

In this project, I consider complex algebraic varieties arising from the representation theory. Geometrically defined operations induce meaningful algebra action on the cohomology (or the K-theory) ring of such varieties. The most prominent examples are generalized flag varieties equipped with Hecke-type algebra action and Hilbert schemes of points in a plane with Heisenberg algebra action.

In both cases the considered variety can be decomposed into locally closed subvarieties. For generalized flag varieties we have Bruhat decomposition, and for the Hilbert scheme there is Białyński-Birula decomposition. To each stratum one assigns certain element of cohomology called its characteristic class. It turns out that in both cases algebra action allows to compute classes of more complicated strata from the classes of simple ones. The class of the smallest cell is easy to compute. The study of algebra action is an effective way to determine classes of more complicated cells.

The main part of this grant proposal is concerned with a characteristic class called stable envelope. It derives from geometric representation theory. It is defined axiomatically, as the only element which satisfies some set of conditions. The proof of its existence is nonconstructive. In general it is very hard to find such an element. In the case of homogenous spaces stable envelope coincides with another characteristic class called the twisted motivic Chern class. This equality allowed to prove that a Hecke type algebra action may be used to construct element satisfying the axioms of stable envelope. It provides an effective way to compute stable envelopes.

Stable envelope depends on a parameter called slope. Its construction using Hecke algebra action was initially known only for the special value of this parameter. Recently, in a joint work with A. Weber we generalized this result to the case of an arbitrary slope. Several results based on the inductive construction of stable envelopes are still proved only for the special value of slope. In this project I plan to extend them to the general case.

The Kirwan map allows to describe stable envelope for homogenous variety using a simpler object: rational function in many variables. Such a function is not unique, yet sometimes (e.g. in type A) there is a distinguished choice of representative. This choice is also connected to Hecke type algebra action. One of the goals of the project is to find geometric meaning of this action and distinguished representatives for an arbitrary semisimple algebraic group.