Quantum geometry and BPS states

National Science Centre – OPUS research project

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Since a discovery of Quantum Mechanics around a hundred years ago, we have known that various physical quantities, for example energy, which characterize objects on microscopic scales, are neither continuous nor deterministic. Instead, such quantities take discrete values and obey probabilistic laws. Furthermore, for around a half of the century, we have known the Standard Model – a quantum theory that describes behavior of elementary particles. A new spectacular confirmation of the Standard Model was a detection of the Higgs boson in CERN in 2012. In view of the great success of quantum theory, physicists believe that not only quantities such as energy, but also space and time are quantized, and governed by a hypothetical theory of quantum gravity. A few formulations of quantum gravity have been proposed, however we still have no means to confirm these ideas experimentally. Nonetheless, such ideas are also actively studied in certain simplified models, which are broadly referred to as quantum geometry; not only do they illustrate the essence of interesting physical phenomena, but also inspire and turn out to be deeply related to various branches of contemporary mathematics.

The aim of this project is to understand models of quantum geometry that are related to so called Bogomol'nyi-Prasad-Sommerfield states (referred to shortly as BPS states). BPS states can be thought of as particle-like objects that arise in supersymmetric quantum field theories, i.e. supersymmetric versions of theories similar to the Standard Model. What makes such models even more interesting is that at the same time they enable to illustrate and understand various strongly coupled phenomena, which are too difficult to analyze in Quantum Chromodynamics, i.e. a part of the Standard Model describing strong interactions. In particular, in one class of theories that we are going to study in this project, an infinite family of complicated BPS states arises as bound states of a few basic states, whose interactions are encoded in a graph called a quiver. Moreover, recently the PI with collaborators discovered that for appropriate choices of quivers, at the same time such theories characterize properties of knots (such as those that are tied on a piece of rope), while counting of associated BPS states is captured by combinatorial and statistical models. This leads to an intricate and interdisciplinary web of connections between quantum geometry, strongly coupled phenomena, mathematical theory of quivers, mathematical knot theory, and statistical models. In this project we are going to study such a web of dualities. On one hand, we will reveal properties and calculate various amplitudes in supersymmetric theories, among others by taking advantage of recent developments in mathematical quiver representation theory and knot theory. On the other hand, reinterpreting physical phenomena in mathematical language, we expect to be able to formulate deep mathematical conjectures, which presumably could not be deduced without identifying the above mentioned web of dualities and physical intuition. Above all, we expect that the results of the project will reveal important properties of quantum geometry, and will contribute to formulation of realistic theories of Nature. The project is conducted by the PI and his research group, in the Chair of Quantum Mathematical Physics the PI is also a head of, at the Faculty of Physics, University of Warsaw.