## ABSTRACT FOR THE GENERAL PUBLIC


The notion of *groups* is a classical concept that occupies a central role in pure mathematics. They are used to describe symmetries of a given object.

In addition to mathematics, groups and symmetries are also of primary importance in natural sciences. In physics, groups aid us in understanding the Universe, and are crucial in string theory, for instance. In chemistry, symmetries can be used to distinguish various spatial isomers and describe their different chemical properties. In biology, groups are widely used in the study of capsids ("shells") of viruses.

This has led to the development of the branch of mathematics called *group theory*. In this branch we study, from a mathematical viewpoint, the algebraic structures created from all symmetries of a given object. The project focuses on research in group theory.

While some objects, such as molecules or capsids of viruses, have finite groups of symmetries, there are also mathematical entities that have infinitely many symmetries, such as an infinite 2-dimensional grid or the whole 3-dimensional space. This motivates the study of infinite groups. It turns out that such groups can be studied not only as algebraic objects, but also as geometric ones, in particular using the notion of *curvature* that has its roots in the area of differential geometry (flat objects, such as a tabletop, are thought as having zero curvature, and a sphere, such as the surface of a ball, has positive curvature). Groups that are of particular interest are the ones which are associated to *non-positively curved* (NPC) spaces; this has been a major motivation in the field of study known as geometric group theory—a relatively young branch of modern mathematics, born 40 years ago in the intersection of abstract algebra, differential geometry, combinatorics and algebraic topology.

In this project, we also focus on the study of a class known as *biautomatic groups*, motivated by the theory of *automata* residing in the area of computer science. Roughly speaking, these are groups for which there is an algorithm (or an "automaton") to list symmetries lying in those groups, and some other automata describing the algebraic relationship between those symmetries. Such a description allows us to perform efficient computations in biautomatic groups, giving a better understanding of this family.

While the definition of biautomatic groups lies primarily in the intersection of algebra, combinatorics and computer science, geometric methods such as non-positive curvature have proved to be extremely useful in their study. In particular, many groups of symmetries of NPC spaces are known to be biautomatic, whereas a plethora of group-theoretic properties that are common in the study of non-positive curvature are also known in the context of biautomatic groups. Nevertheless, in some cases such a relationship is not as straightforward, which has been recently been exemplified by a construction of I. J. Leary and A. Minasyan of non-biautomatic groups of symmetries of non-positively curved spaces, answering a long-standing open question in geometric group theory.

The aim of the current research project is to compare and contrast the classes of biautomatic groups and groups of symmetries of NPC spaces. On the one hand, the property of being biautomatic could give insight into various groups of interest, such as *Artin groups*—a family that has been widely studied in geometric group theory, yet is not well understood apart from certain specific subfamilies. On the other hand, we also aim to develop new methods for working with all biautomatic groups, finding further parallels between them and techniques used for studying non-positive curvature. Such results could deepen our understanding of symmetry, as well as its role in mathematics, natural sciences and computer science.