Intrinsic ergodicity and uniqueness of equilibrium states of dynamical systems

(Abstract for the general public)

The natural and social sciences use mathematics to obtain models, i.e., descriptions of phenomena occurring in the surrounding world written using equations and functions. *Dynamic systems* are models that describe processes that change in time or space. Their analysis helps to understand and predict the evolution of the processes they depict.

The best possible situation that can occur in the study of dynamical systems is when we can perform a classification of all possible results of simulations of such processes with any accuracy. In this way, we get to know all possible outcomes of the evolution of the process that may happen in the future. In a sense, such a classification is the ultimate goal of the theory of dynamical systems, and more precisely, in creating this theory we try to describe all possible dynamical systems.

Of course, in full generality we will never succeed, but in some cases we can say something. In order to achieve something, however, we often have to restrict our attention to specific families of dynamical systems.

A common and effective way of obtaining useful methods for classifying dynamical systems is the following procedure: first, we must specify when we consider two dynamical systems to be the same (in this case, we will also say that the two systems are *isomorphic*). The two most important types of isomorphism are the concepts of *topological coupling* and *theory isomorphism*. Second, we need to find some properties of dynamical systems that do not change when moving from one system to a system isomorphic with it. Such properties are called *dynamic invariants* and usually reflect different types of dynamical system behavior and complexity. In the best possible case, these invariants can be expressed as a single number assigned to a given system. Note that if the value of a certain invariant is different for given two dynamical systems, these systems cannot be isomorphic.

One of the most commonly used dynamical invariants is *entropy*. It provides a measure of the disorder occurring in a dynamical system by quantifying the exponential growth rate of the number of initial states that can be recognised within a certain accuracy as time goes on with increasing resolution. Generally speaking, entropy measures how much *complex* the dynamics of the phenomenon under study is. The *topological entropy* of a dynamical system examines how many different orbits we are able to observe if we have a device with asymptotically perfect resolution capability. The *Dynamic entropy* of the invariant measure is greater the more complicated the dynamics of the system is when we restrict ourselves to a large set (relative to this measure). It is known that the topological entropy is bounded from below by the *entropy* of each invariant measure and can be approximated arbitrarily well by the entropies of such measures. Measures whose entropy has a maximum value for a given system are called *measures of maximal entropy*.

The main theme of the project is the study of necessary and sufficient conditions for the existence and uniqueness of measures of maximal entropy and similarly defined equilibrium states. Having obtained the existence and uniqueness we will study the properties of the measures so specified. Learning the properties of these particular invariant measures helps in understanding the dynamics of the phenomenon they describe.