

Quaternions are generalization of complex numbers, where we allow three imaginary numbers i, j, k such that

$$i^2 = j^2 = k^2 = ijk = -1.$$

They can be seen as 4-dimensional space $\mathbb{H} = \mathbb{R}^4$ over the real numbers equipped with a special action - multiplication. This multiplication is defined by the above condition where we identify:

$$1 := (1, 0, 0, 0), i := (0, 1, 0, 0), j := (0, 0, 1, 0), k := (0, 0, 0, 1).$$

An n -dimensional manifold is an object such that near any of its point it looks like \mathbb{R}^n . An example of such an object is a sphere. In this project we are interested in $4n$ -manifolds whose tangent plane at each point can be identified with \mathbb{H}^n . Depending on a precise technical mathematical condition about the properties of this identification, such manifolds are called quaternionic manifolds, hypercomplex manifolds, quaternion-Kähler manifolds or hyperkähler manifolds. These manifolds play an important role both in mathematics and theoretical physics.

The aim of the project is a comparison of two type of constructions of hyperkähler and, more generally, quaternionic manifolds, called Feix-Kaledin type construction and c-map. The first of them uses, so called, twistor theory and therefore is implicit, which means that we only know that the structure on considered manifolds exists and can deduce some of its properties, but we are not given explicit formulas. The second one was first discovered by physicists and is completely explicit - we have exact formulas for the structures. These construction, despite having some differences in initial data and properties, have also some striking similarities. Our aim is to check if, up to some extent, they can be seen as equivalent and, if not, to investigate the nature of these similarities. As a result we will increase our understanding of the constructions and will obtain new examples of quaternionic manifolds.