The parity problem for cellular automata.

(Abstract for the general public)

“The Whole Is More Than the Sum of Its Parts”

Aristotle

A cellular automaton is a system consisting of individual cells located next to each other. Each automaton cell can be in one of the possible states. The most important feature of the automaton is that the cells interact with each other. They evolve in parallel in discrete time steps according to state update functions (i.e. local rules): the state of the cell is changed according to the rules of how the new state of the cell depends on its current state and the state of neighboring cells. This feature has made cellular automata a popular tool for studying many properties in complex systems.

Consider transferring data between two devices, that is, sending a message consisting of zeros and ones. One form of checking for errors in the message that may occur as a result of transmission is parity checking. An extra bit is added to the message so that the number of bits in the data chunk is always odd or always even. The parity of a sequence of bits of length N is a global quantity that can be efficiently computed using a global counter in O(N) time. But is it possible to find parity without using a global counter but using cellular automata, i.e. having a local rule that works in a selected neighborhood?

Our project is to be a significant contribution to the study of the parity problem of cellular automata.

Thanks to the methodology we have developed, which we have been using for several years, and our preliminary research, we are sure that the goal of the project is achievable. The basis of our working method is the intertwining of theoretical work with computer experiments. Initial theoretical considerations are followed by computer simulations. The results of these studies allow for setting the direction in the search for hypotheses and the selection of mathematical tools.