Description for the general publics

The research project has two main aims. The first one are solutions to Einstein scalar field equations of Lichnerowicz type:

$$-\Delta_g u + hu = fu^{2^*-1} + \frac{a}{u^{2^*+1}}, \quad u > 0$$

and more general, including

$$-\Delta_g u + hu = f(u) + \frac{a}{|u|^p u}$$

and

$$-\Delta_g u \pm Vu = f(u).$$

These equations are considered in the Riemannian manifold (M, g), which can be treated as e.g. a timespace. The variational considerations so far concerned usually problems on compact manifolds. Our aim is to extend methods and results to a wider class of manifolds. Solutions of such an equations and the description of their properties may be useful to understand original Einstein equations, which descripe the dependence between a spacetime geometry and the distribution of matter in this timespace. Einstein equations are intensively studied, in particular there are attempts to explain the acceleration of the expansion of the universe.

The second aim are solutions to the Schrödinger equation with a singular potential of the form

$$-\Delta u + V(x)u - \frac{\mu}{|x|^\alpha} u = f(x, u).$$

This equation appears in e.g. quantum mechanics and nonlinear optics. For example, in nonlinear optics, f is responsible for the polarization of the material. When the parameter μ is sufficiently small, the equation has been already studied. However, there exists a "limiting" number μ^*, on which the equation changes its behaviour. We plan to carefully analize this problem exactly in the case $\mu = \mu^*$.