Representations of algebraic semantics for substructural logics

Logic is about reasoning, but reasoning, apart from having to be logical, is also always physical. The processing device can be human brain, or it can be an information processing machine sometimes known as “artificial intelligence”. Intelligence or not, physical substratum is required. This simple fact immediately rules out classical logic, for classical logic is not sensitive to resources. They can be used in any order: if you can get A from B and C, you can get A from C and B. Redundant resources can be added: if you can get A from B, then you can get A from B and C. A single resource can be used multiple times: if you can get A from C and you can get B from C, then to get A and B you need only one C, not two copies of it. These features are known as the *structural rules* of *exchange*, *weakening*, and *contraction*, respectively. They are well and good in mathematics, but when one deals with implementations, day-to-day reasoning, or artificial intelligence, they fail.

Substructural logics come to the rescue. In them, structural rules do not apply, at least not all, not fully, or not always. Substructural logics have been successfully used in linguistics (Lambek calculus), in approximate and probabilistic reasoning (many-valued logics), in content-sensitive reasoning (relevant logics, non-Fregean logics), in engineering (fuzzy logics), and in computing (intuitionistic logic, linear logic).

Classical logic provides an absolute standard and rules the ideal mathematical world, but in the messy everyday world of applications she can be and is helped by her younger substructural sisters. They are many an varied, so they need to be studied, analysed and classified, so that we could put them in appropriate drawers and keep them there for future use. But substructural logics themselves are *mathematical objects*, so they must be studied mathematically using classical logic. Indeed such studies form a good part of mathematical logic, and quite a few important results have been established. One general result is that semantic correspondents of substructural logics are *residuated structures*.

But residuated structures are a vast class, and a lot of them are not well understood. Hence the need to represent them as something more familiar. This situation is commonplace in mathematics, and several *representation theorems/theories* for common mathematical structures exist.

In this project we study representation theory of a large class of residuated structures, called *perfect residuated lattices*. We use the tools from universal algebra and category theory.