Von Neumann factors from compact quantum groups (abstract for the general public)

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The theory of groups is a very well established field of mathematical research with applications in most other branch of mathematics and physics. The reason for such amazing versatility of this theory is that it describes and studies the notion of *symmetry*. Hence any problem which involves any kind of symmetry transformation (of geometric, algebraic or other nature) will necessarily be amenable to the methods of group theory.

Among the many subdisciplines of group theory a special place is reserved for so called *harmonic analysis* which is in part devoted to the study of "unitary representations of topological groups", i.e. realizations of abstract groups (e.g. symmetries of physical theories or problems) as linear operators on Hilbert spaces which preserve the scalar product ([Mau1968, Dix1977]). Coincidentally, in physics Hilbert spaces also describe states of quantum systems and hence the theory of groups and their representations has become an integral part of quantum mechanics and quantum field theory ([Wey1950, vNeu1955]).

It is at this point where the notion of a *quantum group* (on operator algebra level) enters. Quantum groups are objects generalizing locally compact groups in the framework particularly suited to the study of unitary representations. Although a precise description of what a quantum group is lies beyond the scope of this abstract, let us satisfy the reader's curiosity by stating it in rough terms: a quantum group is an object generalizing the concept of a locally compact group in a way analogous to how quantum mechanics generalizes classical mechanics. In particular, the (commutative) algebra of functions on a group is replaced by a (non-commutative) algebra of operators on a Hilbert space.

The project "Von Neumann factors from compact quantum groups" is aimed at a thorough study of the examples in which the quantum group is "compact" (a technical notion, see e.g. [NeshTu2013, Wor1998]) and the associated algebra is of very particular type, namely it is a von Neumann algebra with trivial center (so called *factor*). The expected results include construction of uncountably many examples of such quantum groups exhausting all possible factors of certain class (so called injective type III_λ) factors and classification of these examples using new invariants of somewhat technical nature.

References

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