Structure and algorithms for graph classes defined by transduction-closed properties

Jakub Gajarský

The main motivation for this project comes from considering algorithmic metatheorems. These are results of the form: Every problem of a certain type can be solved efficiently on a certain class of inputs. Such results are very useful, because they allow us to establish the existence of efficient algorithms for many problems with a single result. This idea has turned out to be very successful in the case when the problems we consider are expressible in a certain logical language and the inputs come from a class of well structured graphs. In this case, algorithmic metatheorems have the following form: Every problem expressible in a certain logical language can be efficiently solved on a certain class of graphs. Once such a metatheorem is proven, to show that a particular problem has an efficient algorithm, it is enough to write its definition in the logical language in question, and this definition then can be turned into an efficient algorithm.

The most famous metateorem is Coucelle's theorem from 1990, which says that every problem expressible in monadic second-order (MSO) logic is solvable in linear time on classes of graphs of bounded treewidth. While this theorem is strong and useful, its main shortcoming is that classes of graphs of bounded treewidth are rather restrictive, and there is no good way to extend it to classes of unbounded treewidth (one notable exception would be graph classes of bounded clique-width, but those ale also quite restrictive). A natural way to attempt to overcome this is to consider the more restrictive first-order (FO) logic; this is the logic we are interested in in the project. The FO logic is substantially weaker than MSO (meaning that we can express fewer problems), but allows us to prove metatheorems for much richer classes of inputs (much more general graph classes). In the past 25 years, the question on which classes of graphs one can solve problems expressible in FO logic has been extensively studied. Initially, the focus was mostly on classes of sparse graphs. This was a successful line of research which culminated in 2014 by the result of Grohe, Kreutzer and Siebertz, who have shown that every problem expressible in FO logic can be solved efficiently on all sparse (nowhere dense) classes of graphs.

Shortly after, the attention has shifted to more general classes of graphs which are not necessarily sparse. Of particular interest are classes of graphs which are closed under transductions (these are graph transformations which are based in logic and which can be used to define new graph classes from old ones). This has been a successful research direction which has seen a lot of interesting progress recently, and which is expected to be active in the coming years.

In the project we will study structural, combinatorial and algorithmic problems related to graph classes which are closed under transductions. In particular, we will be interested in identifying and analyzing various structural properties which graphs from such classes can have. Such properties can then be used for example to show the existence of suitable decompositions for graphs from a given graph class, providing algorithms for computing them and in developing logical tools which will allow us to prove new algorithmic metatheorems.