Proposal Title: Dynamical systems and descriptive complexity

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Abstract for the general public:

A couple of centuries ago, it was believed that most physical phenomena could be described with absolute precision. All you need to do so, is to come up with the right model and the right mathematical tools. Differential equations seemed to be a perfect machinery to perform this task.

At the end of the 19th century this viewpoint started to shift. Poincaré was the first to realize that sometimes the right formula is impossible to discover; in mathematical terms it means that some differential equations are impossible to solve analytically.

To overcome this difficulty, the qualitative study of dynamical systems emerged. Mathematicians changed the paradigm and tried to describe how models behave qualitatively, and not give exact, quantitative predictions of what can happen. By the early 70s of the last century, it became apparent that certain models are extremely complex, for example, weather models. Even the short-term precise predictions of the weather with these models poses a great challenge also for the best computers, and long-term forecasts are practically impossible.

Nowadays, we have developed many tools for studying the complexity of a particular dynamical system. But our quest to explain complexity is far from over and we keep searching for new ways to understand it.

In this project, we take a look at the complexity in dynamics from the viewpoint of another area of mathematics, descriptive set theory. The latter theory has close connections with definability and computability theory. Very broadly speaking, it tries to show how complex certain mathematical notions are, when we define these notions rigorously using logical sentences.

Using tools and notions offered by descriptive set theory, we are going to attack several problems aiming at not only describing the complexity of a single dynamical system, but trying to assess the complexity of whole families of dynamical systems.

Working and connecting with mathematicians from several universities around the globe, we expect to obtain discoveries in this direction. We hope that our results will not only lead to publications, but also provide us with tools to tackle many more related problems expanding our understanding of real-world processes.