Popular Science Abstract

The project concerns research in real algebraic and analytic geometry. The basic objects in real algebraic geometry are regular maps between algebraic sets in real affine spaces. The components of such maps are given by regular functions, which are rational functions with nonvanishing denominators. Working mainly with smooth algebraic sets, we focus on the problem of approximation of infinitely differentiable maps by regular maps. According to the classical Weierstrass approximation theorem, such an approximation is always possible for real-valued functions or, more generally, maps with values in real affine spaces. However, the approximation problem is very difficult for most other target spaces. For example, it remains an open problem whether every infinitely differentiable map between unit spheres can be approximated by regular maps. It is often convenient to consider the so-called k-regulous maps, which are rational maps (possibly having points of indeterminacy) with k-times continuously differentiable extensions, where k is a nonnegative integer. It is known that k-regulous maps are more flexible than regular maps and, in general, have better approximation properties.

Approximation by regular or k-regulous maps are hard problems because the standard methods known from differential topology completely fail in the algebraic setting. As a related topic, we also consider k-regulous vector bundles, which form an intermediate class between algebraic and topological vector bundles. Within the framework of the project, we raise some natural conjectures and problems regarding approximation of maps and properties of k-regulous vector bundles, which will be thoroughly investigated.

An additional area of our interest is the theory of arc-analytic functions, i.e. real-valued functions on a real analytic manifold that become analytic after being composed with parametric real analytic arcs. Arc-analytic functions need not be continuous, let alone analytic. Our goal is to characterize among arc-analytic functions those that are analytic or become analytic after being composed with a locally finite sequence of blow-ups with smooth analytic centers.

The project is a continuation of a research program that has attracted the attention of the broader mathematical community and was presented by the Principal Investigator, jointly with Krzysztof Kurdyka, as an invited lecture at the International Congress of Mathematicians 2018 in Rio de Janeiro.