Suppose that we have n points on the plane, no three in a line, and we want to know how many pairs of them one can join by lines without building a triangle, which means that there are no three points $a,b,c,$ so that the lines ab, ac, and bc are in our family. The answer to this question was given by Mantel in 1907: to maximize the number of lines one should split the set of points into two equal parts S and T, and join each point from S with each point of T. This result is considered one of the earliest theorems in modern graph theory and its proof is rather simple.

Now let us consider a similar problem, when we try to draw as many as possible triangles with vertices at n given points in such a way that there are no four points a,b,c,d for which four triangles abc, abd, acd, and bcd are in our family. The answer for this problem was conjectured by Turán in 1941, but despite many efforts we are very far from verifying it. Another natural question can be stated as follows: suppose that we join n points not by lines, but by arrows, in such a way that each point is a tail for more than $n/3$ of them. It is true that then we have to create either a pair of opposite arrows ab, ba, or a directed triangle with arrows ab, bc, ca? This conjecture, due to Caccetta and Häggkvist, is one of the major open problems in the theory of digraphs.

The main objective of the project is to prove structural results as well as to develop new tools which could shed some light on a number of similar, hard yet elementary stated, problems on graph, hypergraphs and digraphs.