Partial Differential Equation (PDE) constrained optimization is at the core of wave-based imaging problems that appear in several fields from medical sciences to earth and environmental sciences. In this methods, some receivers are located at the boundaries of the object which is to be imaged (for example at the surface of the Earth in the seismic imaging methods) and record the wavefield that has been produced using suitable sources at appropriate locations. Imaging is the process of determining the physical properties of the medium, that appear as the coefficients of the governing PDE, from the recorded data. The conventional approaches to solve such problems are based on optimization. This means, one introduces some function (e.g., L2 norm) that measures the mismatch between modeled and observed data and then try to find their minimizers. The found minimizers are then set to be the solution to the imaging problem. However, such mismatch functions are nonconvex and thus have multiple local minima, thus conventional gradient-based optimization methods face limitations to solve the problem. That means, in order to find a solution that is close to the “true” solution, one needs to have a “sufficiently good” estimate to be used as the initial guess. In this case, we say that the optimization method is not robust with respect to the initial model.

One of the approaches for solving the problem of local minima and finding globally optimal solutions is using global optimization methods. This approaches, however, are restricted to small size problems and face limitations for solving real-world imaging problems. An interesting and practical approach to solve nonconvex imaging problems is to extend the search space. The basic idea is to relax a nonconvex problem into a sequence of convex subproblems whose solution converge towards the global solution of the original (nonconvex) problem. The generated convex subproblems are then solved efficiently by using well documented numerical methods available for efficient solution of convex optimization problems, such as least squares and linear programming problems.

In the last decade, different methods under various names have been developed to increase the degree of the convexity of wave-based imaging problem. In a group of these methods, extended methods, the data residuals are mapped into a set of parameters that are artificially added to the optimization problem. In this case, data fitting is no longer a problem because they can be matched even with erroneous velocity models. These artificial parameters are then penalized by updating the image such that at the convergence point we arrive at the original problem. The two main categories robust of these methods apply extension in the image domain or in the data domain, each having its own advantages and disadvantages. Despite very promising results obtained by these methods, many open questions and clarifications still remain.

This project aims to answer to those questions as much as possible and develop efficient methods to solve the inverse problem in wave-based imaging without requesting a good initial model. Specifically, 1) we will combine the properties of the methods based on extension in model domain and data domain to develop an efficient and robust algorithm having the advantages of both; 2) we will develop an efficient and robust method for waveform inversion based on multipliers method designed for the real media with elastic properties; 3) we want to determine the theoretical connection between the extended formulations and the standard Newton’s method.

Achievements of this project will be significantly useful to wave-based imaging community in general and seismic imaging in particular. Besides better understanding of the mechanism of the extended methods in increasing the robustness of the inverse problem the methodological developments in this project will allow geoscientists to have an efficient tool for various studies of the Earth’s structure and better image the challenging subsurface models in both exploration and crustal scales that are otherwise impossible.