Structural and algorithmic properties of hereditary graph classes

The concept of NP-hardness and NP-completeness, fundamental to modern theoretical computer science, is based on the observation that a large number of combinatorial computational problems, most of which abstract challenges in everyday life, are equally difficult in terms of computational complexity. In other words, an efficient algorithm solving one of them entails the existence of similarly efficient algorithms for all of them, which is considered highly unlikely. Within the concept of NP-hardness, the term "efficient" formally means that there exists an algorithm solving the considered problem in time bounded by a polynomial in the input size on any input. Importantly, the only allowed measure of the input is its bit-size or, for convenience, closely related measures such as the number of vertices and edges of the input graph for a graph problem. However, multiple examples show that such a high-level point of view hides too much: if other (mostly structural) parameters are allowed in the running time-bound, one can discover vast areas of tractability that are hidden from the "only input size" point of view.

This highlights the importance of understanding the *structure* (as opposed to only *size*) of the input. In this project, we focus only on *graph problems*, where the input consists mostly of (sometimes annotated, usually undirected) graph and the structure refers to the structural properties of the input graph. Graphs model networks, be it computer, road, electricity, or social networks, just to name a few.

This leads to the following question: for a fixed NP-hard graph problem, which structural restrictions on the input graph make the problem easy (or easier)? In this project, we focus on a wide family of graph problems that ask for a sparse subgraph with specific properties. The most notable problem from this family is MAXIMUM WEIGHT INDEPENDENT SET (MWIS for short). Namely, given a graph with vertex weights, find a subset of vertices of maximum total weight that is independent (sometimes called stable), which means that no two chosen vertices are adjacent. For example, one can model units of a wireless network as vertices in the graph where edges represent possible interference; then, an independent set corresponds to a choice of a subset of units that do not interfere with each other.

How one can methodologically study these isles of tractability? One can ask about the complexity of a given problem (e.g., MWIS) when the input is restricted to some fixed graph class. This consideration brings us to the following research question:

> Which hereditary graph classes are sufficiently well-structured to allow more efficient algorithms for classic combinatorial problems, such as MWIS?

Here, *hereditary* means that a graph class is closed under vertex deletion; this is one of the weakest regularization assumptions that allows us to avoid obscure examples.

Uncovering a deeper structural and algorithmic theory for MWIS (and related problems) is the main goal of this project. Recent advances suggest that the classes of H-free graphs (graphs excluding a fixed graph H as an induced subgraph) for H being a path, a subdivided claw, or a forest of those, are good testbeds for a general algorithmic theory for MWIS. We expect that the developed framework will have a genuine impact far beyond the classes of considered H-free graphs. We aim at a general methodology for solving combinatorial problems such as MWIS that is adaptative concerning restrictions on the input graph.

Furthemore, we seek for applications of the developed ideas in other areas, lying between algorithmic and structural graph theory: the notion of χ-boundedness or the Erdős-Hajnal property, as well as combinatorial algorithms in perfect graphs and related graph classes.