DIMENSION-FREE ESTIMATES IN HARMONIC ANALYSIS AND BEYOND IT

The research proposed in the project lies in the mainstream of harmonic analysis. Harmonic analysis (or Fourier analysis) is a branch of mathematics that grew out of the theory of Fourier series. Its main idea is a decomposition of the studied object onto a sum of simpler components. Methods of harmonic analysis find broad applications not only in mathematics but also in everyday technology: in computed tomography, data compression (MP3 and JPEG formats), or in signal processing (for instance in radio waves). Our project concerns so called dimension-free estimates in harmonic analysis and applications of these estimates and methods of proofs to other issues in mathematics.

A common theme in modern mathematical analysis is the boundedness of various linear or sub-linear operators in a d-dimensional space. Usually, even if these operators are bounded, we have no control on the bound when d grows. One of the main goals of this research project is to delve into situations where, despite d increases, we do have a control on the bound for the investigated operators. This phenomenon is known as dimension-free estimates. In our project such estimates are mainly studied for natural Hardy-Littlewood averages connected to high-dimensional convex sets.

Similarly, the aspect of dimensional dependence is also important in some problems of number theory. Consider for instance the task of finding the number of solutions within natural numbers of the equation $x_1^2 + ... + x_d^2 = N^2$. This issue is known as Waring's problem and related topics have been studied by mathematicians for centuries. The number of solutions of Waring's problem depends not only on N but also on the dimension d. Asymptotic number of these solutions for very large N is known for a about 100 years. However, it is not clear what happens when N is of medium or small size compared to d. Another goal of our research project is to study this problem, in particular using methods from harmonic analysis.

Recent years have witnessed a dynamical development in Fourier analysis of Boolean functions. This area of mathematics finds applications in a number of topics in computer science, for instance those related to quantum computing. Emerging questions often concern dimension-free estimates for various operators connected with the discrete Laplacian. In our project we also study such problems.

The topics described above may sound highly abstract. Yet, they are related to mathematical fields that are closer to applications - those that study models with a large number of parameters. Such models are present in data analysis and machine learning and are related to the so called 'curse of dimensionality'. Models with a large number of parameters are also important in probability theory and mathematical statistics in which case the relevant phenomenon is called 'concentration of measure'. Applications of these models include: deducing the macroscopic behavior of systems composed from a large number of particles in physics, speed up in measurements in medicine, and fast data processing in data science. What lies at the heart of applications of these theories in practice are estimates - independent of the dimension of the space (understood as the number of parameters). Such estimates are exactly those that we will study in our project from the point of view of harmonic analysis. Thus, because of a similarity between methods and goals, our research project may be also helpful in these areas.