The fluctuations of Bose-Einstein condensates are a long studied phenomenon with a thorny history in the history of quantum mechanics and ultra-cold quantum gases. Erwin Schrödinger in 1944 was perhaps the first to note that the predictions of the contemporary theory of the ideal gas in the grand canonical ensemble yield unphysically large fluctuations. With time, it was observed that in the case of condensate fluctuations, different statistical ensembles are not equivalent even in the macroscopic limit, a remarkable situation with few parallels in statistical mechanics, that continues to be widely debated to date. An example of the difficulties of the seemingly simple problem of a uniform Bose gas is the behaviour of the shift of the critical temperature in an interacting gas which was studied by many groups in the 1990s which came to wildly different conclusions that even disagreed on the sign! e.g. one work by Kerson Huang famously and wryly summarised its conclusions as “This disagrees with all existing results analytical or numerical”.

While the initial theoretical questions mentioned above for single component gases in three-dimensional space have now been resolved, experimental studies were not done for reasons of technical difficulty. That is until an experiment made at the University of Aarhus in Denmark in 2019 that finally was able achieve the necessary accuracy in preparation of atomic clouds with a systematically controlled number of atoms to measure the predicted fluctuations.

This has reset the topic and posed a new challenge to theorists: now predictions for other kinds of ultracold Bose gases such as dipolar or multi-component “spinor” gases have become fair game for experimental verification. Remarkably, we find that few such predictions exist. Meanwhile, in collaboration with the a group at the Centre for Theoretical Physics, Polish Academy of Sciences and the Aarhus experimental group, we have recently developed a numerical technique that can be adapted to these other kinds of systems, including for macroscopic sized clouds, which was very difficult in the past. Therefore *the objective of the current project is to obtain an understanding of the global condensate fluctuations in dipolar and multicomponent quantum gases, and to determine the conditions under which the statistical ensembles become equivalent again as interactions are introduced between the atoms.* These are very fundamental but poorly explored quantum phenomena that have become very experimentally relevant recently.