Description for the general public

For the last two decades, the interest in partial differential equations (PDEs) with *non-local operators* has grown rapidly and the ever wider group of researchers has begun to study the subject. The specific feature of non-local operator is that its evaluation on a function u at any point of the space depends on values of u on the whole space. These types of operators have been investigated in quantum physics for a long time. Recall here relativistic Schrödinger operator. However, the increasing interest in the non-local operators came from the fact that numerous scientific publications revealed that in the large part of physical, biological, chemical, and mathematical finance models the substitution of classical operators by non-local operators in related PDEs leads to the solutions that better describe the phenomena. Since it became clear that PDEs with non-local operators play a vital role in models of natural sciences, the systematic studies over this class of PDEs have flourished. In the last decade, a great effort has been made to develop the theory of PDEs with fractional Laplacian which is a model example of non-local operator. A lot of papers have been written on this topic, and very interesting methods have been developed. However, most of these techniques works only for the specific operator - fractional Laplacian - and every time there is a need to change the driving operator in the problem, the whole work has to be repeated.

Fortunately, non-local operators are natural in the theory of Markov processes and have been investigated by probabilists since 60' of the last century. It is well known that there is a duality between Markov processes and operators which generates Markov semigroups. This is a very wide class of integro-differential operators which, in particular, includes fractional Laplacian. This duality provides completely new tools for investigating PDEs with non-local operators. The purpose of the present project is twofold. Firstly, we aim to solve, by using probabilistic and analytical tools, some open problems in the theory of PDEs with integro-differential operators. Secondly, we want to further explore the relation between PDEs and stochastic analysis to search new links that may lead to new interesting methods and results, both in PDEs and stochastic analysis.