Abstract for the general public

Linear algebra in orbit-finite dimension

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The starting point of this project is the solvability of orbit-finite systems of linear equations. We call a system of linear equations to be orbit-finite under permutations of variables, if the system is invariant under these permutations, and there are finitely many variables and equations up to these permutations of variables. We explain this concept using the following example.

Consider any infinite set A. Consider an infinite set of variables $\left\{ x_{(a,b)} \;\middle|\; a \neq b \in A \right\}$, i.e. each variable is indexed by a pair of distinct elements from A. Consider the system of equations

$$x_{(a,b)} + x_{(b,a)} = 1,\ a \neq b \in A.$$

Take an arbitrary permutation π of A. We can use π to permute the variables by simply mapping the variable $x_{(c,d)}$ to the variable $x_{(\pi(c),\pi(d))}$, for all $c \neq d \in A$. This permutation of variables induces a permutation of equations which sends the equation $\big(x_{(c,d)} + x_{(d,c)} = 1\big)$ to the equation $\big(x_{((\pi(c),\pi(d))} + x_{(\pi(d),\pi(c))} = 1\big)$, for all $c \neq d \in A$. Notice that, although the variables and equations get permuted, the set of variables and the system of equations remain the same. Moreover, there are finitely many variables and equations up to these permutations. In fact, for this example there is only one variable and only one equation up to permutations of A. Hence, we call this system of linear equations to be orbit-finite (and in this case a single orbit of linear equations).

As our contribution, we expect to settle the questions of decidability of several variants of orbit-finite linear equations. We also expect that by the end of our research, we will have built a robust theory of orbit-finitely generated vector spaces. Finally, we believe we will make significant progress towards settling the question of decidability of reachability of Petri nets with data, a long standing open problem in this area.