Geometric properties of sequences of Sobolev homeomorphisms and homeomorphisms of bounded variation

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Let us imagine a small ball of plasticine. It can be easily *deformed*, i.e., its shape can be changed so that plasticine is not torn apart (it is therefore deformed in a continuous way) and so that it is possible to go back from the deformed to the original ball without again tearing any plasticine apart. Mappings satisfying these three properties - continuity, existence of an inverse mapping and continuity of this inverse - are called *homeomorphisms*. They describe in a natural way deformations of different materials: stretching of a rubber band, bending of a metal plate and crushing a plastic bottle, which are objects of study in the *elasticity theory*. Although homeomorphisms are introduced to students of mathematics as a basic tool in topology, they have become important in mathematical analysis as well. Especially, if they are equipped with some extra properties.

In mathematical analysis, the most desirable property of mappings is the existence of a derivative, because a lot of physical phenomena is studied with the help of differential equations. A term which is both extremely handy and useful (since much more general than the classical derivative) is the *weak derivative*, which is possessed by *Sobolev homeomorphisms*. Questions about geometric properties of convergent series of such mappings form the core of this project. Let us fix a set and observe how it is transformed by the mappings which are close to the limit map. The following question comes to mind: since the mappings are close to one another and the limit map, are their images also similar? The aim of my research is to show that it is indeed so if some additional assumptions are satisfied. One of the most crucial assumption is the *Luzin N property* stating that a mapping cannot produce something from nothing, that is (in a mathematical language) that sets of measure zero are mapped onto sets of measure zero. A common-sense approach suggests that if this assumption, so natural when it comes to describing a real deformation, is not satisfied, then the sequence of images of mappings can behave in an unexpected manner. My plan is to illustrate this intuitive claim with examples and to show sharpness of the chosen assumptions.

There are multiple reasons why it is worth to better understand properties of Sobolev homeomorphisms. One of them is the development of elasticity theory mentioned above. In this area deformations are studied with calculus of variations methods which lead to the question about limits of sequences of homeomorphisms. Unfortunately, the limit mapping does not need to be a homeomorphism (for example, it can turn out to be discontinuous or not invertible). On the other hand, recent research has shown that limits of homeomorphisms inherit in some sense properties that are typical for continuous and invertible mappings. Problems discussed within this project correspond to this area of research as well. Moreover, I intend to use methods concerning Sobolev homeomorphisms and their limits to study properties of sequences of *homeomorphisms of bounded variation*, which possess slightly worse differentiability properties.

While conducting this research, I will use the classical methods of analysis and topology as well as advanced results which have been recently published. Results obtained within this project are going to be communicated over international conferences and published in one or two papers. To this aim, a few short-term visits to leading research institutions excelling in solving problems concerning homeomorphisms in Europe (Prague, Naples) and USA (Syracuse, Pittsburgh) are planned.