# Groups of Manifold Transformations: dynamical and probabilistic point of view.

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Over the last four decades group actions on manifolds have deserved much attention by dynamical systems people. Recently some probabilistic approach has gained in importance also. In the project we are interested in these two aspects: dynamical and probabilistic of groups of manifold transformations.

When a group of transformations on some manifold in $\mathbb{R}^d$ is given, we may ask whether the action of this group is minimal (there are no closed invariant subsets for this group). A natural question that arises in this setting is ergodicity, that is, the nonexistence of invariant sets except for those having zero or full (Lebesgue) measure. Despite some remarkable recent progress even the simplest case of 1–dimensional spaces (a circle or interval) has not been yet completely understood.

In the study of abstract groups the notion of distortion was introduced. Informally speaking, an element $f$ in a group $G$ is distorted if the word length of $f^n$ grows sublinearly in $n$. In this case it is said that the translation length of $f$ vanishes. Lately distortion in some transformation groups has been intensively studied. Having gained an insight into distortion in groups of transformations on the circle, we would like to ponder over the problem of the existence of distorted elements in other frameworks.

Random dynamical systems become more and more important. This is probably due to some interesting and surprising results obtained in recent decades. In the second part of the project we are aimed at studying random walks on finitely generated groups of transformations (the so called random transformations). In fact, we equip a given group of transformation with some probability measure and consider a process (random walk) corresponding to random selection of transformations from the group. Tracing the action of these random transformations on the manifold where they are defined, we come to a Markov chain. We will be looking for some natural conditions sufficient for the existence of a unique invariant measure for this Markov chain. Uniqueness, in turn, assures that a measure is ergodic. In the project we will study some simple random walks on homeomorphisms in $\mathbb{R}^d$ and their ergodic properties. The critical case, that is the case with zero drift condition, we will focus on, surely leads to infinite invariant Radon measures.

In the last part of the project we plan to deal with limit theorems for Markov chains corresponding to random transformations. In particular, a central limit theorem, law of the iterated logarithm and large deviation principle are expected to be established.

From the very beginning groups of transformations have attracted a great attention. The problems we intend to study seem to be fundamental to better understanding of group actions phenomena. Although they appear to be completely theoretical, their connection with statistical physics and computer science has to be emphasized.