

Selected aspects of modelling multicomponent fluids

The project concerns mathematical models of fluids which consist of more than one component. It is a field of fluid mechanics, which is dynamically developed during the last years, both considering mathematical analysis and modelling itself. The subject of our research has also many applications in natural sciences. Apart from the fact that we naturally deal with many kinds of mixtures in everyday life, the similar mechanisms are used to describe different phenomena. For example, the same models are used to analyse mutual interactions between different species of animals. On the other hand, from the mathematical point of view the issue of both modelling and analysis is very complicated. Due to the interrelation between the components, there occur several phenomena, which are not present in the classical models of fluid mechanics. They involve diffusion and chemical reactions between the particular substances. The other interesting problem is also taking into consideration the level of mixing of the components.

The main mathematical tool used to describe the behaviour of the fluid are partial differential equations. At the level of the mathematical description, the interactions mentioned above correspond to a certain modification in the classical equations modelling the motion of a single-component fluid. They also change the character of the equations, which makes the standard methods of analysis inapplicable. Therefore there appears a need for a new method, which will enable to treat these types of systems.

Our research is focused on the properties of the so-called weak solutions. By a weak solution to the differential equation we denote a function, satisfying certain crucial properties of a classical solution, but which has a lower regularity. For example, even though the equation depends on the derivatives of solutions, the weak solution does not have to be differentiable, and sometimes even continuous. In our case it enables us to treat as a solution the density of a fluid with jumps, which would describe the situation where the particular components are not mixing and are contained in the disjointed sets. The proof of existence of weak solutions is also a significant step to prove the more regular ones – first we show that the solutions exist in a weak sense, and then show higher regularity.

The aim of the project is to give a complex answer to questions related to existence of weak solutions modelling the above phenomena and analyse further properties of the solutions, such as uniqueness and regularity. The effect of our work will be extending the current knowledge in that field. We also expect that the developed method will be applicable to different kinds of partial differential equations.