Abstract for the general public

It is undeniable that automatization is one of the most relevant processes in our society, with robots gradually overtaking more and more tasks in factories, public services, or even daily living routines. For "robot" we will understand any mechanical system that has the ability to guide itself autonomously, from humanoid drones to just a simple industrial arm in an assembly lane. Probably the most important function of robots is no other than its ability to modify their environments to accomplish different kind of tasks. Of course, there is the possibility of some changes of said environment, and the robot has to be prepared to face this alterations and be able to mainting its proper functioning. Here is where topology enters in action. Topology is, in simple words, the branch of mathematics that studies shape and transformations without breaking apart or gluing. From the topological point of view, all the possible changes of positions of potencial obstacles in the space of all states of the robot (which will be called *configuration space*) result in equivalent situation, so that allows us to forget the details of those changes, and just focus on the structural properties of the configuration space, making the topological approach easily adaptable.

A sequence of "rules" to direct a robot from one state to another will be called a motion planning algorithm. Of course, the simpler a motion planner is, the better, and the difficulty of the task of constructing said algorithms depends on the intricacy of the configuration space. For this, it is interesting to be able to quantify it somehow and, for that purpose, M. Farber introduced the notion of *topological complexity*. Roughly speaking, this is an invariant through continuous deformations that assigns to every configuration space the number of instructions one has to provide in order to ensure the ability of the robot to operate autonomously.

The objective of the project is to contribute to the study of topological complexity of a particular class of spaces, called "$K(G,1)$-spaces". Also we will adress a particular shortcoming of the classical notion of topological complexity, namely its lack of accountability for the fact that sometimes the configuration spaces come equipped with symmetries, that is, the fact that a robot can present physically different but functionally equivalent states. As an example, consider the following simple case of a mechanical arm

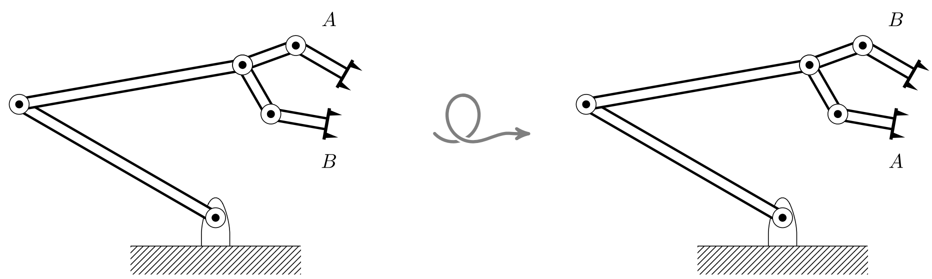

Figure 1: A mechanical arm in physically different, but functionally equivalent states. Figure taken from Z. Błaszczyk and M. Kaluba “Effective topological complexity of spaces with symmetries” Publ. Mat. 62, Issue 1 (2018), 55-74.

For confronting this issue, Z. Błaszczyk and M. Kaluba introduced a version of topological complexity which takes into account said symmetries, and we will devote part of this project to the task of deepening the understanding of that alternative version.