

Physics is the science of mathematical descriptions of physical phenomena. Progress happens when multiple phenomena can be described a common mathematical description, typically called theory, and this predicts new phenomena that can be observed. The paradigm, on which this progress is based, is sometimes called *Reductionism* — more and more phenomena are described by a reduced amount of theory mathematical description. A popular example for this paradigm is the quest for a grand unified theory that describes both phenomena of particle physics (handled by quantum theory) and gravity (general relativity), but there have been simpler examples before in history of science: the unification of electric and magnetic phenomena into electromagnetism, for example. Intuitively, the progress of fundamental physics is such that theories have a certain range of validity (for example up to certain length scale) — a more fundamental theory is then one that can describe physics at an even smaller length scale.

But, many approaches to quantum theory of gravity lead to a problem with that intuition: space and time do not make sense up to arbitrarily small length scales. Depending on the model space and time might be discrete or there is some other mechanism, such that only at big distances it seems smooth and continuous as we know it from day-to-day experience. When employing string theory as a candidate for theory of quantum gravity, this problem can be understood due to the existence of dualities. A duality is the observation that a phenomenon can be understood similarly well in more than one way. An easy example is electromagnetism. In the vacuum electric and magnetic field are interchangeable — the physical phenomena, that can be described by electromagnetism, stay the same when the field switch roles, they are simply described in another way. The so-called T-duality in string theory is mathematically very similar to the electric-magnetic duality but seems physically very different: it roughly states that string theory in a space with length scale R is equivalent to string theory in a space with length scale $1/R$. So, a string theory with a small length scale (a very fundamental one in the nomenclature from above) is equivalent to a string theory with a big length scale (a not so fundamental one). The possible interpretation is that string theory implies a natural minimal distance. Such dualities challenge the paradigms, discussed at the beginning: there might be multiple equivalent mathematical descriptions and the quest for a more fundamental theory (reductionism) might make no sense anymore. Hence, dualities hint at the fact, that the correct map between objects in the physical world and their mathematical representation has not been found yet. Dealing with dualities has been central in the history of physics already. Before a consistent description of quantum mechanics was found, there was a long debate about the wave-particle duality.

This program investigates the role of dualities and the consequences of their existence at the example of so-called sigma models, a broad class of theories that is used in particle physics, condensed matter physics but also as the fundamental description of objects in string theory. Instead as a lucky coincidence, this program takes the dualities seriously and employs a mathematical framework that takes duality as a fundamental assumption. Substantial results that are expected from this new approach, are insights into relations between sigma models in different target space dimension or new ideas how to understand the quantum theory of these models.

As discussed above, the presence of dualities, in particular T-dualities challenges our understanding of fundamental length scales and hence also our grasp of space and time. This geometric approach is expected to give insights into this changed nature of space and time at the fundamental level. The most simple example is, that it allows for the inclusion of the description of non-commutative spaces. These are spaces in which a uncertainty relation holds for the simultaneous measurement of distances in different directions – in contrast to ordinary spaces in which such a uncertainty relation only holds for simultaneous measurement of space and time.