The project proposes research on asymptotic properties (when time escapes to infinity) of very natural evolutionary systems of physical origin, such as:

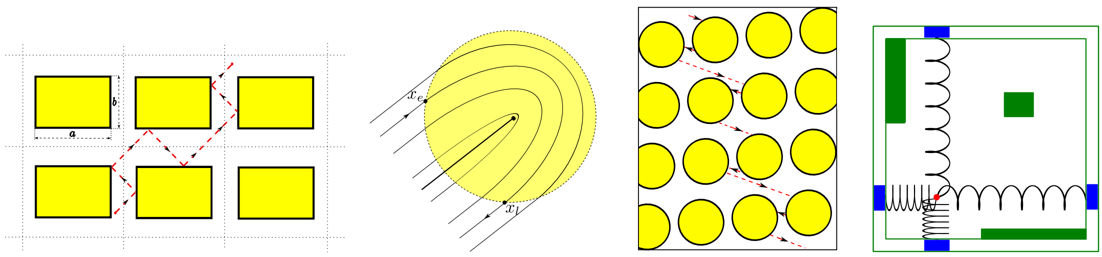
(a) mathematical billiards on infinite tables with piecewise linear boundary;

(b) propagation of light rays scattered in periodic systems of so-called Eaton lenses;

(c) four springs pinned together when the point of pinning reflects off the walls; and

(d) locally Hamiltonian systems on surfaces (motion of electrons on Fermi surfaces).

In this type of systems the object under study changes its position (state) in time, in the case of billiards it is a mass concentrated in a point imitating a billiard ball which moves without friction and rotation and is reflected from the walls of the table; in the system of lenses it is a ray of light passing through successive lenses (a ray of light passing through an Eaton lens reverses the direction of motion); in the case of spring system it is the point of pinning, which moves under the influence of compression and tensile forces of springs and reflections from the walls (we assume that there is no friction in the system and springs are perfectly elastic). The evolution of the described objects in time is usually very complicated, and even when we know exactly the rules of motion and the initial state of the object it is difficult to predict the properties of the object trajectory over a long period of time. Instead of studying individual trajectories, ergodic theory proposes a different



(a) Billiard system (b) System of Eaton lenses (c) Springs

approach. One studies all orbits (almost all) in terms of the frequency of appearance of an object in subsets of the space (called the phase space). In this approach, the basic problem is to determine the so-called invariant sets that is subsets of the phase space in which the trajectories of objects are trapped. If an object falls into such a set, it will not escape from it. If the (possibly smallest) invariant sets are already determined, the next problem is to understand the distribution of trajectories in the invariant set. That is, how often the trajectories fall into different regions of the invariant set. Mathematically speaking, we look for so-called invariant measures. In the case when the phase space is infinite (we have such a situation in systems (a) and (b)), an additional issue is the study of the recurrence of trajectories (infinite return near the initial position), the velocity of scattering and the directions of scattering.

In this project, we propose to study the above problems in the context of systems of types (a), (b), (c) and (d). All these systems have a common feature: the study of their dynamics is reduced to the study of the so-called directional flows on translation surfaces. Such systems are not very sensitive to small changes of initial conditions. They are not hyperbolic systems, for which research techniques are very well developed. The dynamical systems considered in this project are of the parabolic type. For such systems there is no uniform strategy to study the ergodic properties, which poses a great creative challenge to researchers.