Abstract for the general public

In this project, I am going to study mathematical objects called *groups*. The concept of a group is omnipresent in mathematics. It allows to describe various ideas in a structured way. Loosely speaking, groups can be considered as generalizations of geometric symmetries like axial symmetry or point reflection. Groups find numerous applications in almost all branches of mathematics, from algebra to differential equations. The presence of groups is also broadly witnessed in other areas. For example, they can describe models in particle physics and molecular chemistry.

I plan to investigate groups using their *cohomology*. Studying groups by means of cohomology is motivated by trying to *represent* them in a convenient way. Such representations can be found using a concept of a *group module*. Group cohomology provides an insight on the group structure by means of group modules. More precisely, it allows to describe group *invariants*, that is the tools to test specific group properties. I plan to focus on a *geometric group theory* point of view, namely on certain *rigidity* properties, among which one can include *Kazhdan's property (T)* which, roughly speaking, ensures that symmetries of groups with this property have fixed points. To be more specific, the main subject of my project is studying the phenomena of *vanishing* and *reducibility* of group cohomology with *unitary coefficients*. These phenomena generalize property (T) and can be applied to construct new examples of *expanders* which, in turn, find many applications in both theoretical mathematics and computer science.

I am going to approach vanishing and reducibility of group cohomology by means of a specific sufficient algebraic condition, proved recently by Bader and Nowak. This condition involves the study of a certain *Laplace operator* in the *group ring* setting. This study can be performed efficiently by means of *semi-definite* programming, generalizing the *simplex* method from convex omptimization. This involves computer-assisted *rigorous* proofs. I would like to work with $\mathrm{Aut}(F_n)$ which can be loosely thought of as groups of *symmetries of all symmetries*. It has been recently proved that these groups have property (T) which justifies the effectiveness of the *Product Replacement Algorithm* proposed by Charles Leedham-Green and Leonard Soicher in the end of XX century allowing to generate random elements in groups. The proof of property (T) for $\mathrm{Aut}(F_n)$ involved showing the existence of positive *Kazhdan constants*. The question of the reduciblity of cohomology of $\mathrm{Aut}(F_n)$ remains still open, however. I expect to answer this question and, possibly, provide better estimates of Kazhdan constants for $\mathrm{Aut}(F_n)$.

I also plan to characterize the reducibility of group cohomology with unitary coefficients using the condition of Bader and Nowak – I suspect it shall be a necessary condition as well. Once proved, this will be an exact generalization of the result of Ozawa concerning reduciblity of the first cohomology groups with unitary coefficients (which turns out to be equivalent to Kazhdan's property (T)). This would allow to translate cohomology reducibility to finite-dimensional considerations over group rings.