Horizons and gravitational radiation

The general theory of relativity (GR) has recently celebrated the success of detecting gravitational waves. The shape of the recorded signal corresponds to the signals that, according to GR, accompany the collision of two black holes when it occurs. This was another major clue to the existence of these unusual objects in the Universe of which we are a part. However, both the exact theory of gravitational radiation and the exact theory of black holes still remain incomplete. Even the definition of the angular momentum of gravitational radiation created by the collision of two stars or black holes that have changed resultant velocity after merging into one object is debated. The presence of a positive cosmological constant deprives relativists of the traditional tools constructed for gravitational radiation in flat spacetime. The general theory of black holes even with a zero cosmological constant needs to be completed. We know well and understand certain solutions to Einstein's equations describing an idealized stationary spacetime containing a black hole with perfect rotational or spherical symmetry surrounded by a vacuum, or electromagnetic field. However, it is not clear how generic these solutions are. Hawking has provided arguments about the black hole's "rigidity" and "no hair". These show that, under certain assumptions, other black holes than those mentioned above do not exist. "Rigidity" implies the need for axial symmetry, and "no hair" implies a complete characterization of the black hole by a few parameters. In the vacuum case, only two suffice: total mass and total angular momentum. However, these conclusions were derived with a number of simplifying and idealizing assumptions. Getting rid of these simplifications became a topic of research in the mathematical theory of relativity for many years. They led to the creation of new methods and the development of a whole new field of GR dealing with the global structure of spacetime. Despite the enormous progress of our theoretical knowledge, the proof of the "rigidity" and lack of "hair" of a black hole is still incomplete. The most difficult case is when the ratio of a black hole's angular momentum to its mass is maximal. Such black holes are called extreme black holes. In their case many mathematical arguments break down. As a result, much less is known about the possible types of extreme black holes with full mathematical precision than about the others. The inclusion of a non-zero cosmological constant does not make the situation any easier.

The recently discovered similarity between horizon symmetries and symmetries of asymptotically flat spacetime will enable us to create a method analogous to a tomograph for gravitational radiation. It will allow us to describe the shape of the perturbed black hole horizon through the properties of gravitational radiation at infinity. The approach to black hole theory developed in the current project is complementary to the global theory pioneered by Hawking. Our research focuses on the geometry induced at the surface of a black hole by the surrounding spacetime. A black hole understood in this way has infinitely many degrees of freedom even after taking Einstein's equations into account. This seems to be in contrast with the property of no "hair". We therefore look for additional geometric and physical constraints on the local geometry of the black hole surface that would result in a reduction in degrees of freedom similar to that resulting from the global arguments of the Hawking model. We have found such constraints. The extremality condition takes the form of an equation that is now known as near horizon geometry and studied in the context of many generalizations. For generic, non-extremal local black holes, the condition is a restriction of the algebraic type of the Weyl curvature tensor to the so-called Petrov type D. One of the main tasks of the project is to study these equations and prove or reject the mathematical hypothesis that all their solutions are characterized by the rigidity and the no-hair property. We will also investigate new equations that result from successive orders of expansion of Einstein's equations around a black hole and provide a complete description of the spacetime in its environment. The prospective and comprehensive goal of this part of the project is to build a bridge between the results of black hole surface studies and standard global methods. In particular, the local characterization of the black hole can provide arguments that complement global methods and lead to a complete and mathematically satisfying resolution of the black hole rigidity and no-hair problem.