Reg. No: 2021/43/D/ST1/03244; Principal Investigator: dr inż. Grzegorz Ryszard Serafin

Central limit theorem for general functionals with applications to random graphs

Decription for the general public

Everyone has probably at least once heard of Gauss (normal) curve, which had even appeared on German bank notes before changing currency to Euro. It is related to so called normal distribution, which describes many real life phenomena, as for instance spreading of heat or Brownian motion. One can notice its great importance when dealing with numerous independent experiments producing, as a result, random numbers, which happens for example in multiple throws of a dice. Namely, the sum of the results, after a small manipulation, turns out to be well approximated by normal distribution. The same situation occurs in case of human weight, height, or number of accidents on roads. This rule is known as central limit theorem and plays a crucial role in probability theory and statistics.

This research project is devoted to studying central limit theorem-like situations, where we exchange addition of the results of the experiments by some more complicated operations. At the same time, we examine precision of the obtained approximations. In the first part of the project, we focus on general results and estimate the precision of approximation for as large class of operations as possible. Such results are usually of very complex form, so one of the tasks is to simplify them to the form related to so called *fourth moment phenomenon*, which additionally will give simpler conditions ensuring that the obtained quantities can be indeed estimated by normal distribution.

Another part of the project concerns random graphs. A graphs is a set of points - called vertices - where some of them are connected, i.e. there is an edge between them. Random graphs are graphs with a random structure. Usually, this means that edges appear at random, but one can also consider random coloring of vertices or edges. We will study, among other models, random intersection graphs, where every vertex has randomly assigned attributes and two vertices are connected whenever they share a common attribute. Such construction is a good model of social networks where members belong to some groups, which is nowadays a very common situation due to the development of the internet. The main goal of the research is to approximate number of edges, triangles or more complex objects appearing in the graph, which are crucial in analyzing structure of random networks. We will be also counting monochromatic objects in graphs with randomly colored vertices.

The project is a part of the mainstream of current research in mathematics. It touches the very classical topic of central limit theorem on the one hand, and a modern theory of random graphs on the other hand. The considered problems are important in the theory of probability, statistics and combinatorics, which should attract a lot of attention.