Graph theory is a crucial branch of modern mathematics, important in combinatorics, probability theory, dynamical systems, topology, information theory, computer science and more. The emergence of quantum mechanics led to many "quantum" analogues of these classical disciplines, including quantum information theory on the physics side and quantum probability on the mathematics side. Especially the former inspired the introduction of quantum graphs, whose popularity rose in the recent years, also in the context of pure mathematics. Many difficulties in studying quantum graphs stem from the fact that, just like quantum particles do not have a well defined position, one cannot really talk about their vertices.

When you take a look at quantum graphs from a mathematician's perspective, there are certain basic questions that immediately come to mind. What are paths in a quantum graph? What does it mean for a quantum graph to be connected? How do we distinguish them? What are symmetries of quantum graphs? Do typical quantum graphs have symmetries? Some of these questions are already answered, and some of them we plan to tackle in this project.

One of the challenges we will have to face is to find the correct analogues of familiar notions from graph theory. In some cases the translation is fairly straightforward, nevertheless one does still need to develop completely new tools that could be applied in the quantum setting. Quite often the situation is radically different and we have to put a lot of work just to come up with a reasonable "quantum" analogue. In this project we will have to overcome both obstacles, sometimes at the same time, in order to deliver interesting results.

More precisely, we plan to initiate the study of paths in quantum graphs and related random walks. Inspired by the recent connection between quantum symmetries of graphs and quantum information theory we intend to thoroughly investigate quantum symmetries of quantum graphs, in particular focusing on the properties of typical graphs.

We believe that the results of the project will have applications to quantum information theory, theory of dynamical systems, and probability theory, fostering new interactions between these areas.