Approximation techniques for partial differential equations and calculus of variations

The research objectives of the current project are twofold - spread between the studies in the area of calculus of variations and in the area of partial differential equations. This might at the first glance seem to be an overwhelming task for a person entering the research career, however these two fields are closely connected by using the same tools of approximation. The core of the project lies in development of approximation techniques in different, often nonstandard, function spaces. The basic spaces that can serve as an example for understanding the crucial idea are well-known Sobolev spaces. It is a standard task for undergraduate student to show that compactly supported smooth functions are dense in a Sobolev spaces. The benefit and application of this knowledge is significant. All the calculations that could not be accomplished within the setting of rather unfeasible definition of weak derivatives, easily follow in the setting of smooth functions. Then the step of passing to the limit is a last, but not least, to complete the proof of the fact, which is of our interest. We have intentionally underlined that it is *not the least* step, as this is the moment when crucial difficulties arise. They may come from various factors, including: a complexity of a function space, low regularity of terms appearing in the considered equation or particular form of a minimalized functional. The crucial and the first step of this project is to show that elements of appropriate, non-standard function space may be approximated with better objects (like compactly supported smooth or bounded functions). The second stage of research, however really far from being trivial, is application of this tool for two problems. The first one concerns the question of appearance of Lavrentiev phenomena in the studies of existence and regularity of minimizers of functionals. This is a problem attracting recently a lot of attention. The second problem of interest are partial differential equations: either parabolic or elliptic. The technique which almost always seems to be a remedy for a priori estimates or uniqueness proof is testing the equation with a solution itself. This however is not always possible, exactly because of not appropriate regularity of solution and then the approximation results obtained earlier are of key interest. The function spaces that are considered are related either with a form of a functional or with the differential operator in partial differential equations.