Research project: Lorentzian spectral geometry and its application in physics,
or How to Make a Spherical Universe Look Like an Hourglass

The universe we live in has four dimensions. Three of them are easy to notice and all know about them: height, width and length. Then there's a fourth one, and people often forget about it because it's invisible, but still everyone knows it: *time*. You might be somewhat skeptical about calling time a dimension, but still, it makes sense: after all, we know that we can move forward in time (the future gradually becomes present and then past), and we can conceive also the notion of *going back in time* – which is something science fiction built tons of books and movies around. So, since we can move back and forth in it (at least conceptually), we may regard time as a dimension as well – albeit a weird one. Indeed, we can't see it even though we can see the other three, and while it's easy to go forward in time, it is impossible to go back (sci-fi aside).

In fact, time is somewhat counter-intuitive also from a mathematical point of view. If a child asked a mathematician what is the difference between a universe that has time versus one that only has space, the mathematician would answer that the one without time would look like a sphere, while the one that *does* feature time would look like an hourglass. It is immediate to understand which one is more "natural": if you walked along a riverbed and looked at the pebbles surrounding it, would they look more like an hourglass or a sphere?

Yet, in spite of these considerations, our universe *does* feature time, thereby incontrovertibly decreeing that it is actually *more natural* for time to be there. Behind the confounding blanket of technicalities typical of mathematical physics, what our project really aspires to is to shedding light on this *unnatural naturalness* of time – understanding why time exists and how to describe it in the mathematical framework known as *Non-Commutative Geometry*.

In this modern era in which television, movies and the internet are commonplace, science fiction greatly contributed in popularizing the profound insights of Einstein's theory of General Relativity: that gravity is nothing but the effect of the deformation of space around the sun, the Earth and other celestial objects. To get an idea of this fact, imagine laying a heavy bowling ball down onto your bed's mattress. Of course, the bowling ball will deform the mattress and create a deep valley in it. Now, suppose that there were already other small ping-pong balls on your bed before you laid the bowling ball down. What will they do after you lay down the bowling ball? They will roll towards it. In some sense, the bowling ball is attracting the ping-pong balls. This is gravity in a nutshell.

Mathematically, this means that *gravity is a manifestation of the shape of space*. Physicists have found out that something similar is true also for the other forces, like electromagnetism or the nuclear interactions: all these are themselves *the manifestation of the shape of some intangible mathematical entity* that pervades all space – something like an invisible cloud that permeates the whole universe. And that's not all: the very existence of photons and another whole bunch of particles can be traced back to this "cloud" as well.

Non-Commutative Geometry takes one step further: instead of splitting the universe into space(-time) and the "cloud", it describes one bigger (and stranger) space that is both at the same time, so that the *manifestation of its shape* this time describes *both* gravity *and* the other forces. And it even manages to explain the reason for the existence of some other particles as well, like the famous Higgs boson, that – for an unexpected yet funny series of events – went down to history as the *God Particle*.

However, Non-Commutative Geometry, as it is now, is still too naive, for the universe that it describes is shaped like a sphere, not an hourglass. This is precisely the problem that we want to tackle in our project.

To do so, we will consider an alternative version of Non-Commutative Geometry, called *Lorentzian Non-Commutative Geometry*. The price of having an additional difficult term in the name of the discipline is well spent, for this different version actually manages to describe a universe that is shaped like an hourglass. However, to date this theory only tells us about half of the universe, for there are currently no known methods to compute the formulas for the other half.

A recent paper by N. V. Dang and M. Wrochna explains how to perform such calculations in a very simple case. In this project, we aim to use their approach as a stepping stone to build a more general technique that works also for the more complicated case of our universe.

We will use that technique to try out the several formulas that have been proposed so far to describe the latter half of the universe (since there was no way to compute them, no one knows which ones work yet) and single out the right one.

Finally, we will apply the formalism to our universe, and we will do a phenomenological analysis by comparing the expectations of the model with the experimental data to check whether Non-Commutative Geometry really is suitable to describe our universe, and to discover any new physical processes that the model might expect.