Fluctuations of Bose-Einstein condensate-the next step

The cooling of dilute boson and fermion gases to nanokelvin temperatures initiated a new field of physics: physics of quantum gases. Its best-known phenomenon is Bose-Einstein condensation. This phenomenon, predicted in 1924, was realized in 1995. A dilute gas of bosons, below a critical temperature begins to occupy the ground state of a trapping potential, which is typically parabolic. While textbooks use the grand canonical ensemble to describe the properties of a condensate, this experiment is close to the conditions of the microcanonical ensemble, because the gas neither exchanges heat nor particles with its surroundings. Meanwhile, already Schrödinger noticed that the statistical properties of the condensate population depend on the choice of the statistical ensemble and are absurdly unphysical when described by the grand canonical ensemble. In the last 25 years, fluctuations of the condensate have been intensively studied theoretically. For years our group has successfully participated in these studies. It is worth emphasizing that even for a perfect gas it is very difficult to calculate microcanonical fluctuations for experimentally realistic condensates containing hundreds of thousands of atoms. For 3 years it has been possible to measure condensate fluctuations. It is done by Jan Arlt's group from Aarhus. We cooperate with this group. The result is two joint papers published in Physical Review Letters. There is still a problem that has not yet received a satisfactory solution namely to account for the interactions. Some authors claim that the variance of an interacting gas is larger than that of an ideal gas. Others claim that the opposite is true. In this project, we plan to make significant progress in the study of condensate fluctuations. In our previous work, we use Monte Carlo methods. We have developed two methods to generate a set of points in a suitably defined space using the Metropolis algorithm. In both methods we generate a model of the canonical ensemble. Postselection, the restriction of the set of points to a single energy, produces a representation of the microcanonical ensemble. The first method, based on the classical fields approximation correctly accounts for the modification of the condensate wave function caused by the interaction in the case of harmonic trapping. Just as classical electrodynamics led to ultraviolet divergence for blackbody radiation such divergence occurs also in our case. Approximately correct results are obtained by choosing an appropriate cut-off parameter. So there is a necessity to use the fitting parameter. This is an obvious weakness. Our latest method, the Fock states sampling method, restores atoms to their corpuscular character, eliminating the divergence in the ultraviolet, just as Planck's idea of light quanta cured blackbody radiation. The method is very good for accounting for interactions for a gas confined in a box. It is worse in a harmonic trap where the interacting condensate has a modified wave function. The main goal of this project is to construct a new Monte Carlo method that will have the advantages of both our methods without suffering from any of their disadvantages. The new method, adapted to harmonic traps, consists of two steps. In the first, using classical fields method we find not only the wave function of the condensate, but also a large number of single-particle wave functions, which together form a basis better for the second step than the Fock states basis. The second step consists in using this new basis to build a Metropolis algorithm identical to the one used in the Fock states sampling method. In this way we will calculate the effect of repulsive interactions on the canonical and microcanonical fluctuations of the bosons in the harmonic trap. The procedure will be numerically very complex. We are confident that it will be efficient in one dimension. It will be a major achievement if a condensate of 100 atoms can be described in three dimensions. However, before we proceed to the construction of the new method, a preliminary step will be an important test of our previous method, based on Fock's states. This is because we will calculate the interaction-induced shift of the critical temperature for a condensate in a three-dimensional box.