Abstract for the general public

The topic of the research project concerns different possibilities of applications of the Lempert theory – theory that may be described as the one developed as a consequence of the fundamental Lempert theorem that shows the equality between holomorphically invariant functions in convex domains and that was proven in eighties. The Lempert theorem may be seen as a generalization of the classical Riemann mapping theorem that is the fundamental result of the classical function theory of one complex variable. The Lempert theory evolved and has had and still has impact on a growing number of problems in the analysis of several complex variables and recently in the operator theory.

A special stress should also be put on the role of two special domains (the symmetrized bidisc and the tetrablock) that have appeared recently, delivered interesting phenomena in the Lempert theory and consequently they have had an important role in developing the Lempert theory and they have proven to be interesting in the operator theory.

Within the presented research project a number of applications of the theorem and general open problems was formulated. In principle the problems are very difficult and they go well beyond the level of the PhD thesis (e. g. the problem of the equality of invariant functions in C-convex domains or the characterization of domains admitting the finite universal Caratheodory sets). Nevertheless, additional specials cases of the open problems were presented in the project whose solution would be a good basis for a very good PhD thesis. Moreover, these partial solutions may contribute in future to solving the general problems and they also may be interesting in their own. In particular, it means that the results obtained during carrying out of the project may be a good basis for research articles that could be published in reputed mathematical journals.