

This project involves several branches of mathematics such as algebra, transcendence theory, and logic. Transcendence questions were central already in 19th century (transcendence of e and π). It is rather surprising that we still do not even know whether the number $\pi + e$ is irrational! Schanuel stated in 1960's a conjecture which implies the algebraic independence of e and π , but it is wide open till today.

Much more is known about the functional transcendence. For example, the exponential function is transcendental over the field of rational functions and it is also algebraically independent from this very function composed with itself. A general result about such functional transcendence was conjectured also by Schanuel and proved by Ax in 1971. There are many functional transcendence results of Ax-Schanuel style these days and they have numerous applications. This project will deal with transcendence questions mostly in the case of positive characteristic, where in practise only the functional case can be studied.

In Ax's approach to transcendence problems, differential techniques were crucial. One of the goals of this project is to develop the differential techniques in the positive characteristic case using the appropriate notion of a derivation, which was introduced by Hasse and Schmidt in 1930's. By using these methods, we aim to show new Ax-Schanuel type results.

The other part of this project focuses on the "underlying structures" used by Ax above, that is on fields with derivations, and, more generally, on fields with operators such as automorphisms. We will consider both arithmetic as well as logical properties of such fields with operators. By "logical" here, we mean mainly "model-theoretic". Model theory originates from the pioneering work of Tarski in the 1930's. Nowadays, there are many interactions between model theory, geometry, and algebra. This project fits into such general interactions.

We expect to show new transcendence functional results (mostly in the case of positive characteristic) and use them to obtain diophantine and model-theoretic consequences. We also aim at specifying new topological notions regarding fields with operators and at applying them to find new interesting theories of such fields.