## 1. Abstract for the general public


The program of the project fits into the basic research and aims to develop mathematical knowledge about the methods of Hilbert spaces in two main aspects. These are algebraic methods, including selected tools of harmonic analysis on semigroups, as well as methods derived from the broadly understood theory of measure and integration, including the integration of functions with respect to operator measures. The research program of the project focuses on three research issues in which the above-mentioned methods intertwine. All issues are problems so far unsolved.

The first issue is related to the answer to the question of when a given measure whose values are positive operators on a Hilbert space is spectral, i.e., when its values are orthogonal projections. We expect to get the answers in terms of a finite (possibly the smallest) number of operator moments. The language of operator moments fits perfectly into the modern approach to the second quantization of quantum mechanics proposed, among others, by Kiukas, Lahti and Ylinen in 2006, on the one hand, and on the other hand, it allows for solving problems in pure operator theory, such as the Curto, Lee and Yoon problem concerning subnormal $n$th roots of quasinormal operators. The problem is highly non-trivial as it is intimately connected to the unsolved Arveson's hyperrigidity conjecture from noncommutative Choquet's theory and to the Korovkin type theorems.

Another issue is related to the time shift operator of the modified Brownian motion process, studied by Agler and Stankus in the 1990s. Orthogonal integrals of such operators, known as Brownian operators have a $2\times 2$ block matrix representation with the lower right term being a unitary operator. If we replace the unitary term with a more general operator, then new unexpected questions arise in the theory of Agler and Stankus. One of the natural questions is whether the Brownian type operator with the lower right term being a subnormal operator has a coordinatewise extension to the Brown type operator with the lower right term being the normal operator. This issue refers to contemporary trends that have appeared in the theory of lifting operators to operators more general than isometries.

The last of the three research issues concerns conditionally positive definite operators. The class of these operators evolved while searching for effective criteria for subnormality of bounded operators, in particular composition operators on $L^2$ spaces with affine symbols. Research in this area lasted many decades and resulted in more and more effective criteria proposed by Halmos, Bram, Embry, Lambert and Agler consecutively. The culmination was the criterion based on the concept of positive definiteness in the semigroup sense. One of the issues that we intend to investigate in this project is the problem of similarity of a conditionally positive definite operator to a subnormal operator. This problem is difficult because, as we already know, a conditionally positive definite operator with the spectral radius smaller than one is automatically subnormal, on the one hand, and on the other hand, the class of conditionally positive definite operators contains the collection of 3-isometries which is very distant from the class of subnormal operators ($m$-isometries were introduced by Agler; the spectral radius for such operators is always equal to 1).