The project is situated at the intersection of three mathematical subjects: algebraic geometry, commutative algebra and mathematical physics. Algebraic geometry is the study of algebraic varieties which are sets of solutions of systems of polynomial equations. Commutative algebra studies algebraic structures called rings (sets with two operations addition and multiplication satisfying some natural properties) that have the additional property that multiplication is commutative. Finally mathematical physics is a branch of mathematics, studying in rigorous mathematical terms, problems inspired by physics.

The main aim of the project is to provide constructions, understanding and partial classification for a vast class of new examples of special varieties called projectively normal Calabi-Yau threefolds. Focusing on this type of varieties we are at the intersection of three research areas each of which provides its own tools and motivations and the planned results of the project should have considerable impact on all three of them.

It is a leading problem of algebraic geometry to classify all algebraic varieties. However, algebraic varieties form a so vast collection of objects that there is no hope to classify them in general. Instead, one restricts to special classes of varieties which are important for various reasons. Among the most important types of algebraic varieties one finds so called Calabi-Yau manifolds. Their significance in algebraic geometry follows from the special place such varieties occupy in the classification theory; they are in some sense on the boundary between the better understood class of Fano type varieties and the varieties of general type for which there is no hope of general understanding. For instance, the classification of Calabi–Yau threefolds is nowadays one of the biggest challenges in algebraic geometry.

An additionally reason for focusing onto Calabi-Yau threefolds is the fact that they are used in physical string theory to model the shape of the universe. Roughly speaking string theory postulates that the universe is fibered by tiny Calabi–Yau threefolds in which there are vibrating strings. The behavior of the string on the Calabi-Yau threefolds determines the type of particles that we observe. For that reason understanding Calabi–Yau varieties is crucial for the understanding of the string theoretical model of the universe. The theory of Calabi–Yau threefolds is developed in parallel with that of string theory and as such it is full of conjectures motivated by physics. The most famous is the mirror symmetry conjecture whose mathematical counterpart postulates that Calabi–Yau threefolds arise in pairs having some of their structure interchanged. Some mathematical versions of mirror symmetry have been proven for the easiest to handle types of Calabi–Yau threefolds, so-called complete intersections in toric varieties. Outside this class the conjecture is widely open and motivates the need to find new non-standard constructions of Calabi–Yau manifolds.

By now classical results, any Calabi-Yau threefold admits a special good embedding to projective space whose image is a so called projectively normal subvariety. Projectively normal Calabi-Yau submanifolds have the property that a cone over them, locally around the vertex, is associated to a ring satisfying the property that it is Gorenstein. The structure theory of Gorenstein rings is one of the main problems motivating research in commutative algebra. Taking into account the above it is natural to make an attempt to understand projectively normal Calabi–Yau threefolds, algebraic varieties important for physics and appearing naturally and suitable to be studied by purely algebraic tools.

The idea to approach our problem is first to work not with the whole cone over the Calabi-Yau manifold but with a section of it being supported only in the vertex of the cone, it is a so called Artinian scheme. It appears that a lot of information about the Calabi-Yau manifold is contained in the ring associated to such a section, which is a Artinian Gorentsein ring and to which many classical tools of algebra apply. Starting from that idea, we plan to classify possible Artinian Gorenstein rings arising as sections of cones over Calabi-Yau manifolds in the case in which the dimension of the ambient projective space is not too big. From that classification we want to reconstruct the studied Calabi-Yau manifolds. We expect many natural constructions. Having produced many new examples we plan to use them as testing ground for mirror symmetry and other theories originating from physics.

It is also interesting to observe that the Artinian Gorenstein rings arising in our approach are in one to one correspondence, by a suitable duality, to varieties in projective spaces defined by one polynomial equation, so called hypersurfaces, which in our case are of degree 4. Via this duality called apolarity the properties of the Artinian Gorenstein ring translate to properties of this hypersurface. In particular, minimal configurations of points containing the Artinian Gorenstein scheme correspond to decompositions of the polynomial defining the apolar hypersurface into a sum of powers of linear forms. Such decompositions for polynomials are of independent interest since they have ample concrete application in computer sciences, for instance in data processing. From a more geometric point of view, decompositions of polynomials as sums of powers are sometimes parametrized by nontrivial algebraic varieties whose study is also very interesting and will be the object of investigation in the project.