ALGEBRAIC COMBINATORICS AND GEOMETRY IN REPRESENTATION THEORY

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In algebraic combinatorics one uses either algebraic methods and results to solve combinatorial problems, or combinatorial ideas to study algebraic phenomena. In this proposal, we propose a plan to solve certain problems in representation theory, which are of algebraic nature, using geometric objects and combinatorial techniques. In representation theory, the aim is to study difficult algebraic objects via simpler ones such as finite dimensional vector spaces with symmetries.

1	3	5	7
2	6		
4	8		
9			

Figure 1. A standard Young tableau of shape (4,2,2,1).

One of the most classical examples where we use combinatorics to understand algebraic objects is that of standard Young tableaux, which are arrays of left aligned boxes, stacked one below the other in weakly decreasing lengths, filled with numbers from 1 to some fixed number n so that they increase along rows and columns. By recording the lengths of the rows of our tableau we form a sequence of weakly decreasing positive integers, also known as a *partition* which we call the *shape* of the tableau. A finite-dimensional *representation* of the symmetric group on n letters S_n is a finite-dimensional vector space on which S_n acts by symmetries. It is not surprising that the isomorphism classes of finite-dimensional irreducible representations of S_n are given by partitions whose parts add up to n. What is surprising is that the number of standard Young tableaux of a fixed shape equals the dimension of the corresponding representation. Young tableaux are actually used in many contexts: to describe characters of representations of general linear groups, intersection theory of Grassmannians and Schubert varieties, and enumeration of graphs embedded into surfaces. The combinatorics of Young tableaux and similar objects describe not only difficult objects in representation theory, algebraic geometry and enumerative combinatorics. Indeed, their reach extends to high-energy physics, probability theory, statistical mechanics, and many more areas of current research today.

In the current proposal, we aim to apply modern algebro-geometric results and combinatorial methods to study q-multiplicities of representations of reductive groups, which can be thought of as groups of continuous symmetries. Our methods are of similar flavor as the introductory example above. We aim to define statistics on generalizations of tableaux to give long-sought formulas for the aforementioned q-multiplicities, as well as describe other beautiful objects in representation theory explicitly via combinatorics of tableaux and related objects.