The spectrum of a matrix is the set of its eigenvalues. It constitutes one of the most fundamental analytical invariants of a matrix. More generally, in the case of operators on infinite-dimensional Hilbert spaces the spectrum holds valuable information about the operator.

For operators such as the Laplacian on a discrete group certain spectral properties carry crucial information about the group itself. In particular, if there is a gap in the spectrum around 0 then the group has Kazhdan's property (T), a powerful rigidity property that has strong implications for actions and algebras associated to the group or graphs constructed from its finite quotients.

In recent years a new approach to proving such spectral properties has emerged. It is algebraic in nature and involves showing that certain elements in the group ring are positive, by which it is meant that they are finite sums of squares. Such a condition is finite-dimensional and can be attacked by numerical methods.

The PI jointly with M. Kaluba, D. Kielak and N. Ozawa have used such an approach successfully to prove that the group of automorphisms of the free group on at least 5 generators has Kazhdan's property (T), answering in the affirmative a long-standing open problem.

This new approach to proving spectral phenomena by means of algebraic and numerical methods lends itself naturally to generalizations to higher cohomology of groups, a classical theory of algebraic invariants of groups and spaces. The project focuses on studying such generalizations. The main goal of the project is to use this novel approach to prove new spectral phenomena for the cohomological Laplacian and other, related operators.

The results will have interesting applications, including new vanishing theorems for higher cohomology and new rigidity results for groups together with their applications to approximability of groups; constructions of higher-dimensional expanders, which are families of simplicial complexes with exotic geometric properties; and finally, constructions of new types of counterexamples to certain versions of the Baum-Connes conjecture, which is a well-known problem in higher index theory with roots in the classical Atiyah-Singer index theorem.