ONE-PARAMETER DEFORMATIONS IN SYMMETRIC FUNCTIONS THEORY

Symmetry is one of the most interesting and beautiful phenomenons that have fascinated humanity since ancient times. The abstract world of mathematics has long been influenced, both by the outer world as well as the problems raised up by physicists. It is therefore perhaps no surprise that highly symmetric mathematical objects turned out to be the most important ones. A **symmetric polynomial** is a good example of such an object. Its definition is simple — it is a polynomial with the following property of symmetry: no matter how we interchange its variables, the polynomial will remain unchanged. For example:

$$\begin{aligned} f(x_1,x_2,x_3) := x_1 \cdot x_2 \cdot x_3 &= f(x_1,x_3,x_2) = f(x_2,x_1,x_3) \\ &= f(x_2,x_3,x_1) = f(x_3,x_1,x_2) = f(x_3,x_2,x_1) \end{aligned}$$

is a symmetric polynomial, but

$$f(x_1,x_2,x_3) := 2x_1 + x_2 + x_3 \neq 2x_2 + x_1 + x_3 = f(x_2,x_1,x_3)$$

is not. It turns out that some symmetric polynomials are surprisingly remarkable– **they appear naturally in many different fields of mathematics and physics**, and their beautiful and rich structure leads to many important discoveries in those fields. A prominent example is given by *Schur polynomials* — originally discovered as the building blocks of the representation theory of the general linear group, they were quickly recognised also as a basic tool in the representation theory of the symmetric groups, in the intersection theory of Grassmannians and Schubert varieties, or in the theory of matrix integrals. More recently, fascinating connections between enumerative geometry, integrable hierarchies, complex geometry and intersection theory were established by various researchers, including Kontsevich, Mirzakhani, Okounkov and Witten prized by the Field medals, the highest distinction in mathematics. These discoveries tell us, roughly speaking, that a certain important object (a so-called *tau function*) encoded by Schur polynomials has very different faces: it can be described by simple objects easy to manipulate with (**combinatorics**), it can be also described by a system of equations (**integrable systems**), and finally it can be described by some complicated geometric shapes (**geometry**).

Many important models in mathematics and mathematical physics can be realized as special cases of a wider model with an additional deformation paramater, often called "quantum" parameter. It turns out that Schur polynomials naturally fit into this framework — they are specialization at $b = 0$ of so-called *Jack polynomials*, which are symmetric polynomials with an additional deformation parameter b. These polynomials, introduced in the seventies of the 20th century, have already been proved to be interdisciplinary by mathematicians and mathematical physicists. Nevertheless, a one-parameter analog of the important ties between various fields constituted by Schur polynomials is not known to exist. An important piece of evidence suggesting that the aforementioned net of connections might be only a shadow of a more general picture is given by a conjecture, posed in the nineties, which gives a precise (**combinatorial**) meaning of this one-parameter deformation. Despite many partial results proving some special cases, it eluded all the efforts and remains open.

Motivated by this important conjecture, we were able to show recently that, indeed, a combinatorial face in the classical case is just a projection of a more general, one-deformed picture, which we established. The main goal of this project is **to solve the aforementioned long-standing open problem** as well as to **discover and study other links** that hold at the "quantum" level of one-parameter deformation. In particular we plan to establish hypothetical **integrability** and **geometry**. This will give a new, global understanding of remarkable connections between various fields of mathematics and mathematical physics. As a result we plan to use these newly established connections **to obtain results in seemingly unrelated fields of mathematics**.