

One of the deepest principles in mathematics, going back to Descartes' *La Géométrie* in 1637, is the equivalence between geometry and algebra. Geometric objects can be studied by algebraic tools, and algebra can be studied by thinking about it geometrically. *Noncommutative geometry* is the study of geometry when the associated algebra is noncommutative. Far from being just a generalization of conventional geometry for its own sake, noncommutative geometry is in fact forced on us by the basic principles of quantum mechanics, which require physically observable quantities to correspond to noncommuting operators on a Hilbert space.

In another vein, the origins of graph theory are linked to the well-known problem of *The Seven Bridges of Königsberg*, which asks whether it is possible to devise a walk through that city while crossing each of the seven bridges over the Pregel river once and only once. It was the solution of this problem by Euler in 1736 that laid the foundations of graph theory.

Nowadays, graph theory is a highly developed branch of mathematics that studies graphs as mathematical structures that model pairwise relations between objects. A typical graph comprises a set of vertices along with a set of edges connecting certain pairs of these vertices. Edges can be symmetric (undirected graphs) or asymmetric (directed graphs), and can have further properties such as colors or weights. Graph theory has numerous applications in many other disciplines, including computer science, biology, chemistry, physics, linguistics and social sciences. In mathematics, graphs are used not only in combinatorics, but also in geometry and algebra. Algebraic methods can be used to study graph properties and graphs can be used to define interesting algebras such as path algebras or graph algebras.

Directed graphs, also known as quivers, are a formidable interface with abstract mathematics. They feed imagination with the tangible and encode the structure and combinatorics of a wide variety of mathematical objects. Through a universal construction of path algebras, they facilitate a classification of finite-dimensional algebras. Better still, they have led to the construction of many interesting  $C^*$ -algebras and greatly enhanced the understanding of key families of examples coming from quantum groups and noncommutative topology. In particular, directed graphs yield new tools and intuition for analysing operator algebras, complementing what can be understood through classical topology or geometry. Indeed, graph  $C^*$ -algebras have gained the reputation of “operator algebras that one can see”.

The fundamental goal of this project is to solve problems in noncommutative geometry by developing new and innovative tools using or extrapolating highly effective theory of operator algebras that one can see. The project is focused on research objectives whose scope ranges from the combinatorics of directed graphs and colored graphs to the quantum-geometric picture of Leavitt path algebras, Kumjian–Pask graph algebras, and their  $C^*$ -completions, i.e. graph  $C^*$ -algebras and higher-rank graph  $C^*$ -algebras. All these objectives have the common denominator of back-and-forth relations between the structure of such graphs and their algebras. Due to the ubiquity of graphs throughout mathematics and beyond, we expect that achieving our research objectives will be of interest not only to specialists, but also to a wider community of mathematicians and scientists.