Holomorphic dynamics, fractals and thermodynamic formalism. Given a transformation f of a space X increasing distances, iterating action of f leads small sets to large ones (dynamic "escalator"). So long time behaviour under the action of f provides an insight into the local structure of X. We mainly consider f preserving angles (conformal) so preserving shapes, changing scales.

We work mainly in the Riemann sphere S^2 with a holomorphic f (preserving angles, except critical points, where the derivative Df is zero) acting on an open domain U, and limit sets of this action. If $U = S^2$ then f is rational (ratio of two polynomials) of degree at least 2, and one considers Julia set $X = J = J(f)$, compact f-invariant with *chaotic* dynamics, and its complement $F(f)$, called Fatou set. This $F(f)$ has at most finite number of periodic connected components and components of their preimages for f^n. These periodic ones are attracted to attracting periodic orbits inside or parabolic or *singular domains with elliptic dynamics* (irrational rotation in appropriate holomorphic coordinates): Siegel discs or Herman rings. $F(f)$ can be empty, so $J = S^2$; otherwise $X = J$ is nowhere dense, usually *fractal.* This word was introduced and popularized by Benoît Mandelbrot in 1970-ties, for shapes with Hausdorff dimension HD strictly bigger than the topological one. HD(X) roughly says how many small balls are needed to cover X. Though for all polynomials, J is known to be fractal (except z^d and Chebyshev), for rational f (except also $J = S^2$) the problem is unsolved and needs a deep insight in the structure of Julia set in presence of singular domains. Thus proving HD$(f) > 1$ for connected J, not being an analytic curve, is a primary objective of the project.

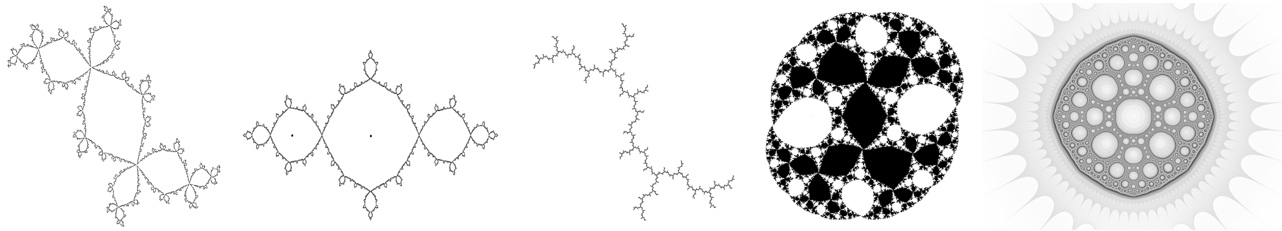

FIGURE 1. Julia sets: rabbit $f(z) = z^2 - 0.123 + 0.745i$, basilica $f(z) = z^2 - 1$, dendrite $f(z) = z^2 + i$, basilica mated with rabbit $f(z) = \frac{z^2+c}{z^2-1}$ for $c = \frac{1+\sqrt{-3}}{2}$ with $J(f)$ being the boundary between white and black, Sierpiński-Julia carpet $f(z) = z^2 - 1/16z^2$ i.e. boundaries of Fatou components do not touch each other.

A method to study a fractal is to distribute masses on it. For $J(f)$ use an f-invariant equilibrium measure (state), that is maximizing entropy plus integral of a potential. The maximum is called pressure (or free energy, as in equilibrium statistical physics). One distributes an equilibrium m_t on small discs B according to their diameters in a power $t = \alpha$ for which these measures sum up to 1 (for different t a normalizing factor is used). The potential is $-t \log |Df|$, the pressure denoted $P(t)$; notice that $|Df^{-n}|$ are roughly equal to these diameters, where n are times of arriving to large scales. $1/t$ plays the role of temperature. α occurs (usually) to be HD(J). So the potential and pressure are called *geometric.* • Assumed the existence of m_t we shall study statistical properties for the sequences $\phi \circ f^n(z)$ for m_t-almost all z and for observables ϕ. Cesaro averages and large deviations are on the stage. • We shall study (a spectrum of) HD's of the sets of points z with given Lyapunov exponent $\lim_{n\to\infty} 1/n \log |Df^n(z)|$ as Legendre transform of $P(t)$. • We shall study open domains U with Riemann parametrization by unit disc and relate its boundary behaviour in presence of dynamics f, with the fractal properties of ∂U without dynamic escalator. Asymptotic variance appearing, parameterizes them. • Pressure via periodic orbits with geometric weights will be studied (related to prime number theorem) and accumulation of them at non-linearizable periodic orbits or boundaries of (hairy) Siegel discs. • A combinatorial version of Riemann map and coding by rays, are geometric coding trees with their infinite branches γ. They provide a strong method to transport measures from the symbolic space to J. The accumulation sets of γ's are worthy to be better understood. Also relations with Markov partitions and graphs Γ with f extended from J being the boundary of Γ, so methods of geometric group theory can be used. • We shall investigate branched coverings of S^2 *coarse expanding* on X, generalizing rational functions, considering quasi-symmetric conjugacies. • Non-uniform hyperbolicity related to weak recurrence of critical points, will be applied. • We shall attack the problem HD$(J) > 1$ for f-invariant Jordan curve near the unit circle for **real** perturbations of z^d, and the problem whether Hausdorff dimension of hyperbolic sets is the stable and unstable sum.

The project is organized in challenging problems and fields to be investigated, mature enough for breakthrough solutions, opening new horizons. It links dynamics with complex analysis, geometric group theory, measure theory, geometric analysis, topology, probability, and may apply in statistical physics, modeling of chemical and biological processes and even classical mechanics and astrophysics.