Elżbieta Drozdowska

Relationships between algebraic, modal and many-valued quantum logics

The aim of this project is to examine the relationships between selected algebraic, modal and many-valued quantum logics.

An algebraic quantum logic is an algebraic structure (called an orthomodular lattice) which is abstracted from the Hilbert space formalism of quantum mechanics and which is believed to properly represent the relations between quantum experimental propositions. Those experimental propositions contain information about states of quantum objects, e.g. „The spin of object x is 1/2". The orthomodular lattice is believed to play such a role for quantum logic calculus like the Boolean algebra plays for classical logic. The orthomodular lattice is not a Boolean algebra – it is weaker because it doesn't have the distributivity property, only the weaker orthomodularity property.

A modal quantum logic is an interpretation of the orthomodular lattice in terms of modal logic – i.e. with the use of Kripkean semantics and the possibility and necessity operators.

A many-valued quantum logic is an interpretation of the orthomodular lattice in terms of many-valued logic. The need for such an interpretation may be justified using Aristotle's example of „there will be a sea battle tomorrow". What can be said about its truth value before tomorrow comes? If one is a proponent of two-valued logic, one will say that statements about future do not belong to the domain of two-valued classical logic. If one is a proponent of some non-classical logic, one may say that it is possible, probable, or that they have some degree of belief that it will become true. It may be claimed that the situation is analogous to that of quantum experimental propositions. According to what quantum mechanics tells us, the proposition „the value of observable O is α" is neither true nor false before the observable is measured. It is only probable, and the degree of probability is given by quantum mechanical calculations. Only after the measurement is performed, the infinite-valued logic „collapses" to two-valued logic – in an analogy to von Neumann's collapse of the wave function during measurement.

Algebraic quantum logic began with the publication of Birkhoff and von Neumann's paper *The Logic of Quantum Mechanics* in 1936 and in the 1950s it boomed into a wide and vivid field of research. But interest in the use of many-valued logic in quantum mechanics ceased sometime in the late 1950s. What is interesting is that after fuzzy logic has emerged in 1990s, a new interest in many-valued quantum logics has emerged: Łukasiewicz many-valued quantum logics have been studied e.g. by Dalla Chiara, Giuntini and Greechie and by Pykacz. As the title of Jarosław Pykacz's paper promises, there seems to be a „*Unification of Two Approaches to Quantum Logic: Every Birkhoff-von Neumann Quantum Logic is a Partial Infinite-Valued Łukasiewicz Logic"*. In what sense are the two unified? And what is the relationship between the new partial infinite-valued (fuzzy) Łukasiewicz logic and the modal quantum logic derived from the orthomodular lattice?

The main question of this project is what is the relationship between the modal and many-valued logics derived from the orthomodular lattice.

Quantum logic is a theory of the relations between quantum experimental propositions. It is worth pursuing, because exploring its intricacies gives us a better understanding of the meaning of the theory.