SELECTED TOPICS IN APPLICATIONS OF SET THEORY IN FUNCTIONAL ANALYSIS

The modern set theory was initiated at the end of the 19th century by Georg Cantor. Cantor's most important achievements include the introduction of the notion of equinumerosity of sets and the theorem that there are more real numbers than natural numbers, which caused a wide discussion between mathematicians. The existence of a set with of cardinality greater than the cardinality of set of natural numbers, but smaller than the set of real numbers was an open problem called Continuum Hypothesis. The concept of a set was not yet well-defined, which led to many paradoxes such as Russel paradox, and consequently, a complete reconstruction of the foundations of mathematics. At the beginning of the last century mathematicians developed axiomatic of set theory (ZFC theory), which is still the standard nowadays. Over time, it turned out that there are mathematical problems that cannot be resolved in the ZFC theory. Among them was the aforementioned Continuum Hypothesis. The independence of the Continuum Hypothesis from ZFC has been proven by the innovative method of forcing in 1963 by Paul Cohen, for which he was awarded the Fields Medal. The discoveries of that time led to the dynamic development of set theory, as well as its applications in other fields of mathematics.

Our project focuses on the set-theoretic aspects of functional analysis, including the theory of Banach spaces, Banach algebras and C*-algebras. The aim of the project is to solve open problems in the above-mentioned areas of mathematics. We intend to apply new ideas to explore issues that have been intensively researched in recent years. We also plan to use set-theoretic methods in areas where such methods have not been used so far, such as the theory of measure algebras. We expect that our work will significantly contribute to the development of the discussed areas and the strengthening of international relations in the mathematician community.