SINGULAR CONTACT VARIETIES - PRESENTATION FOR GENERAL AUDIENCE

ROBERT ŚMIECH

The problem that is a starting point for our project concerns the classification of a particular type of Riemannian manifolds. The examples of such objects are: curves, surface of a donut or any ball (including Earth) or even the whole Universe. Since 50' it is known that all elemental Riemannian manifolds can be divided into seven classes. One of them - so called quaternion-Kähler manifolds - is especially interesting for the researchers, since all known examples are very particular objects called symmetric spaces. Therefore two mathematicians have formulated a conjecture (unsolved up to present day) that there are no other quaternion-Kähler manifolds. This problem is now known as LeBrun-Salamon conjecture, to honor mentioned researchers.

The most important contribution of LeBrun and Salamon to this issue is the following: they observed that to each quaternion-Kähler manifold one can associate a unique algebraic variety (main object of interest for algebraic geometers) equipped with a contact structure (this notion is quite technical, so we will not try to explain it). Therefore we can try to prove LeBrun-Salamon conjecture by classifying algebraic contact varieties, having at our disposal various tools of different subfield - algebraic geometry. In this way partial progress was achieved (for example proof of the conjecture in some special cases). However we still suffer from the scarcity of examples that limits our understanding of the subject. On the other hand, finding any new example would disprove the conjecture.

The aim of our project is to work around this problem. Namely we want to find a proper generalisation of a notion of the algebraic contact variety (it means we want to decrease our requirements for considered objects), allowing the existence of singular points on it (that is: non-smooth, "with edges") or the degeneracy of contact structure on some small subset, so that some natural modifications of algebraic varieties would still fit in our setting (this is not possible with the usual definition). Of course, we will step outside the field of study defined by the LeBrun-Salamon conjecture, but in turn we will gain considerable freedom of action, that can allow to better understand some properties of contact structure or - in best case scenario - help with constructing a counterexample.