DESCRIPTION FOR THE GENERAL PUBLIC

Graphs are abstract structures whose role is to model interactions between pairs of objects. Such objects are represented by the *vertices* of the graph, while the interactions are represented by the *edges* of the graph each connecting some two vertices. An important family of graphs are *geometric intersection graphs*, in which vertices correspond to geometric objects, such as intervals in a line, arcs of a fixed circle, or disks or continuous curves in the plane, and edges to the pairs of intersecting objects. By considering objects of different kind, we obtain different families of graphs; for example, *interval graphs* are the intersection graphs of intervals in a line, *circular-arc graphs* are the intersection graphs of arcs of a fixed circle, and *string graphs* are the intersection graphs of continuous curves in the plane. It turns out that graphs of this kind are widely used to model various real problems, and efficient algorithms working on such graphs find practical applications in areas like resource allocation or VLSI design. Of course, these graphs are also used to visualize the interactions between various types of objects.

The recognition and isomorphism problems are the two main algorithmic problems considered for every class of graphs. The *recognition problem* for a graph class is to check whether an input graph belongs to this class. For a geometric intersection graph class the recognition problem boils down to checking whether the input graph has a representation appropriate for this class of graphs (for example, as an intersection graph of intervals for interval graphs). The *isomorphism problem* for a graph class, in turn, consists of checking whether two input graphs from this class are isomorphic, that is, whether the vertices of one graph can be one-to-one mapped to the vertices of the other graph so as the edges of the graphs are preserved. For a geometric intersection graph class the isomorphism problem comes down to checking whether the objects of one graph can be bijectively mapped to the objects of the other graph such that the pairs of intersecting and non-intersecting objects are preserved. A search for efficient algorithms solving the above problems constitute the core of algorithmic graph theory from its very beginning. For many graph classes these problems can be solved efficiently by *polynomial-time algorithms*, that is, by algorithms that perform polynomially many operations in the input size. There are known, for example, polynomial-time algorithms recognizing interval graphs, circular-arc graphs, it is also known, based on the generally accepted hypotheses of computational complexity theory, that such algorithms do not exist for string graphs or the intersection graphs of disks. The situation is a bit different for the isomorphism problem – no polynomial algorithm testing the isomorphism of two general graphs has been found so far, and no convincing evidence has been given that such an algorithm does not exist. In some classes of graphs, for example, in the class of interval graphs or circular-arc graphs, the isomorphism problem can be solved by a polynomial-time algorithm; in other classes, for example, in the class of string graphs, this problem turns out to be as difficult as in the class of all graphs.

Recently, the so-called H-graphs, which are intersection graphs of a special kind, have been researched quite intensively. Informally, the class of H-graphs can be defined as follows. Imagine that the graph H is represented in three-dimensional space in such a way that the vertices of H correspond to certain points in this space, and the edges of H correspond to continuous curves connecting the points representing the ends of this edge. Additionally, suppose that the interiors of all the curves are pairwise disjoint. Under this assumption, the class of H-graphs corresponds to the intersection graphs of arcwise connected sets contained in this representation of H (that is, in the union of all the points and all the curves corresponding to the vertices and the edges of H). It turns out that H-graphs generalize many known geometric intersection graph classes, such as interval graphs or circular-arc graphs. As such, H-graphs are the subject of intensive research aiming at generalizing efficient algorithms known for simple intersection graph classes on the wider families of H-graphs, as well as at determining the boundary of "polynomial tractability" for such computational problems as recognition or isomorphism testing. The main goal of this project is to determine these boundaries. It is already known that they lie quite close to the class of circular-arc graphs. Although circular-arc graphs seem to be closely related to interval graphs, they turn out to have significantly different combinatorial and algorithmic properties. In particular, quite number of problems that are solved or admit polynomial-time algorithms in the class of interval graphs, in the class of circular-arc graphs are still open or are computationally hard. The second goal of this project is to further research the structural and algorithmic properties of this class of graphs, and to solve some open problems in this class of graphs.