A dynamical system is a mathematical concept for modeling an object varying in time, using a fixed rule that depends on the current state of the object only, and not, for example, on what happened in the past. Dynamical systems are used in sciences to describe a variety of phenomena, from the trajectory of a bullet shot from a gun, through the behavior of an individual neuron, to the fluctuations in the size of populations of species in a food chain. Solutions to this kind of models, called trajectories, describe the evolution of the states of the modeled item or ecosystem in time. Unfortunately, except for very few cases, like the flight of the bullet, these solutions cannot be typically given by explicit formulas, and analysis of the equations by hand is a laborious task that requires thorough mathematical knowledge. Therefore, computers are often used to get insight into the dynamics more easily. Unfortunately, numerical simulations involve approximating the real solutions, and thus rarely provide mathematically reliable results. Moreover, some important solutions are not stable, that is, a small perturbation may push the trajectory away, and therefore such solutions are difficult to find and to follow.

The main objective of the project is to develop some computational tools for the purpose of automated analysis of dynamics, primarily aimed at providing mathematically reliable results, and to apply them to certain systems. Such a method takes a model of a dynamical system as input, conducts computations, and produces a catalog of important solutions found. It is going to be of great help for scientists who create the mathematical models in order to answer such questions as: "What are safe amounts of fish that one can catch from a lake each season to make sure the population does not die out?"

Automated analysis of dynamics is not a straightforward task. One problem is that computers can only deal with finite data structures, so everything must be described in such a way. The first basic idea is thus to analyze the dynamics in a bounded area subdivided into a rectangular grid like a sheet of checkered paper. Then one can describe the changes of states in a dynamical system as a rule that says which square one moves to from each square in the grid. In order to capture unstable trajectories, we engage Topology. This is a branch of mathematics that primarily deals with the notion of continuity. The reasoning that applies here is similar in spirit to the following fact: If we keep one end of a rope under water (altitude < 0) and the other end in the air (altitude > 0) then the rope must cross the water surface at some point (altitude $= 0$). This principle also works in abstract mathematical setup, except the reasoning is more subtle. As a result of applying this method, one obtains mathematically proved information on equilibrium states, periodic behavior and stable and unstable trajectories.

Another topic concerns the analysis of dynamical systems that exhibit chaotic dynamics. The existence of erratic behavior of solutions, which makes the impression of randomness, even though it is completely deterministic (given by the equation), depends very sensitively on the choice of specific values of parameters of such systems. Estimating the actual amount of such parameters is a highly nontrivial task. Even if one can see in numerical simulations that the dynamics is chaotic-like for a large percentage of the parameters, obtaining a mathematically strict estimate turns out to be unexpectedly challenging. A step forward towards achieving this goal is computing a rigorous estimate of "expansivity" of a map, that is, the rate at which points that are near each other are pushed away in forward time. By combining known and new algorithms with advanced numerical calculations, we expect to develop an effective method that will provide accurate estimates at moderate computational cost.

An especially interesting phenomenon that we also plan to investigate is chaotic itinerancy. It is the case when trajectories in a dynamical system stay for a prolonged time around certain states, and then start wondering in a hard to predict way, until they get attracted by another state. But they do not remain at this state indefinitely, the chaotic wandering suddenly resumes and repeats without any clear pattern. We plan to analyze mathematical models that experience this kind of behavior in order to understand them better. Such systems are used to explain the way our brains work: the chaotic itinerancy corresponds to wandering from one thought to another, and coming up with new ideas. These models are also used in artificial intelligence (neurorobotics) to design spontaneous behavioral switching of robots. It is thus important to get better insights into the various models before we allow real robots act in this kind of an unpredictable way.