Not only is *time* one of the most fundamental quantities present in science, but also it is a concept that we intuitively know from daily life. For physicists—and not only for them—it is a mysterious phenomenon somehow connected with motion and space. Since antiquity, natural philosophers have struggled to comprehend its true nature and provide a strict definition that would satisfy everyone. Since the times of Newton and Leibniz, philosophers' efforts to comprehend these concepts can be seen as a conflict between absolute concepts of space, time, and motion and the relational approach. The revolution in physics caused by Newton seemed to solve the problem of an appropriate definition of time until the beginning of the twentieth century. Physicists forgot about the subtleties related to this topic because everything in physics seemed to work according to Newton's classical dynamics. The first sign of the collapse of the classical notion of reality started when Einstein formulated his theory of relativity. The special and general theories of relativity show that time is just one of the four coordinates that can be transformed between different coordinate systems.

The next great revolution in physics was caused by quantum mechanics, which changed the classical point of view. Every measurable quantity was now described by an operator acting on a Hilbert space. Only the definition of time that was introduced to the quantum mechanic stays the same as in the classical Newtonian interpretation. Time in quantum mechanics is still only a scalar parameter that labels the evolution of the system.

Dealing with these contradictions is one of the crucial aims of modern theoretical physics. Exploring phenomena lying at the boundary of the two great branches of physics: the theory of relativity and quantum mechanics, can be an impulse to develop a better theory reconciling different points of view on the nature of time. Recent publications show a new way of treating time in quantum mechanics, and now time can be understood through measurements of quantum systems serving as clocks. Quantum treatment of clocks no longer invites us to consider them as a background to some dynamic evolution of a given system, but rather as an intrinsic part of a fully quantum setup. As such, it is only natural to exploit the quantumness of the time measurement in order to ask fundamental questions and introduce quantities that may work as a testbed for relativistic and quantum theories.

In our research we plan to study two interesting fenomena – quantum time dilation and indefinite temporal order. The first one emerges after asking the question: What happens when a clock is in a quantum superposition of two different velocities? The other problem involves the possibility that the *order* of two events might be in a superposition, in contrast to the usual notion of superposition that is induced by traditional quantum degrees of freedom, such as spatial ones. We plan to work with these ideas in order to devise schemes that would test the relationship of quantum and relativistic theories, uncovering phenomena that might provide us with further insight into the interlayer between these frameworks.

The problem of understanding the notion of time in the context of the theory of relativity and quantum mechanics requires a dualistic point of view. It requires an understanding of the problem of measuring time in a fully relational way but also understanding how the superposition principle affects the standard definition of time. We plan to split our research into two parts.

1. **Universality of quantum time dilation in a fully relativistic regime**

   In our previous research, we showed that quantum time dilation seems to be universal, i.e. it can be measured independently on the clock model. However, our results were calculated just up to the leading order of relativistic corrections to nonrelativistic physics. Now, we plan to show how the quantum time dilation behaves in a fully relativistic theory. We also plan to extend previous results to the case of a clock placed in curved spacetime. In this way, we would like to confirm that the universality of quantum time dilation holds also in the case of an atom experiencing gravitational time dilation.

2. **Measuring indefinite temporal order by using quantum clocks**

   In this part of our research, we plan to focus on relativistic time dilation to show that a clock simulated by moving atoms experiencing different times due to the theory of special relativity can create an indefinite temporal order of events. This indefinite temporal order would then be used to violate Bell's inequalities for temporal order and prove that time order can be superposed.