Abstract for the general public (in English)

State the project goal, description of research, reasons for attempting a particular research topic and substantial results expected – max. 1 page.

The project is located in the mainstream of Partial Differential Equations and Applied Mathematics, and concentrates on mathematically challenging and practically important classes of nonlinear differential variational-hemivariational inequalities, evolution inclusions and various constrained multivalued problems in infinite dimensional Banach spaces.

The project addresses detailed theoretical questions on well-posedness, classification of inequalities, the properties of the solution set, generic regularity and smoothness of solutions, sensitivity analysis, associated optimal control and inverse problems, etc. Completely novel developments will involve nonautonomous history-dependent and almost history-dependent operators, evolution subdifferential inclusions and their systems, continuous dependence of solutions on major parameter perturbations of the data, and inequalities with nonconvex star-shaped sets of constraints. The project stems from deep mathematical questions involving three common features: the energies are nonconvex, the solutions are nonsmooth with respect to both space and time, and they are subjected to constraints. Most of these problems can not be described by partial differential equations, and some of them do not have a well-defined type. More appropriate formulations are based on two relatively new concepts: the differential variational inequality (introduced in 2008 in finite dimensions) and the hemivariational inequality (initiated in the 1980's). The concept of hemivariational inequality has been originated from variational descriptions of physical processes, and has a precise physical meaning expressing the principle of virtual work or power introduced by Fourier in 1823, and is based on the notion of generalized subgradient of Rockafellar-Clarke for locally Lipschitz functions. From the physical point of view, absolutely pioneering and essential in the understanding of the nature of problems studied in the project will be the distinction between those features of nonsmooth and discontinuous systems that are generic, and those that are researcher-dependent or approach-relative.

There are many reasons motivating implementation of the project, for instance: the use of techniques of nonlinear functional analysis, partial differential equations, etc., in development of rigorous methods in applied science, is insufficient; most open real-life problems are nonsmooth, nonconvex, in general, and involve multivalued operators. The project is driven by the insight that the problems lying at the cutting edge of an area of applied sciences still cannot be successfully solved by existing mathematical tools, and new developments are needed.

The goal of the project is to achieve a significant progress in the development of a general theory of differential variational-hemivariational inequalities. The key issue in the research methodology is to deal with the interaction between weakly converging sequences which approximate the sought solution and various nonlinearities in equations, inclusions, and inequalities. Substantial results are expected on constrained history-dependent differential hemivariational inequalities, generalized quasi-variational-hemivariational inequalities, and coupled systems of history-dependent evolution inclusions. Further, we study a plethora of impressive, sometimes unexpected, applications of new mathematical models with nonsmooth potentials described by differential variational-hemivariational inequalities to contact mechanics of solids (in elasticity, viscoelasticity, viscoplasticity, piezoelectricity, thermoviscoelasticity, electro-elasticity) and of fluids (non-Newtonian fluids, rheological models), and other areas (electrical circuits, economical dynamics, complementarity problems, traffic network problems, biomechanics, medicine, biology, geophysics, etc.).