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Yaglom limits for multidimensional Lévy processes

Popular science summary

The objective of our studies is the behaviour of random processes in a long time horizon under the condition of not leaving a certain fixed set. In the present project we will focus on a particular class of Lévy processes and on sets of the form of a multidimensional cone with its vertex at the origin of the coordinate system. For a visualisation of our problem we may think of a three-dimensional case where a cone is a relatively easy conception and of a random process which moves inside the cone with the restriction that we only consider cases in which it stays inside the set up to the observation moment. To be precise, we will investigate the existence of the so-called *Yaglom limit*, that is the limiting distribution of a properly rescaled process conditioned to stay in a cone. From the probability point of view this problem is related to the existence of the so-called *quasi-stationary distributions* which were intensively studied in the last several dozens of years. When it comes to applications, one may list the financial setting, where, for instance, a given random process may describe stock prices or the insurance company capital and our job is to determine its behaviour under the condition that the prices do not hit the bottom or the insurance company does not go bankrupt. For these reasons, inter alia, the issue of the Yaglom limit has been investigated for various kinds of stochastic processes, both in the discrete and continuous setting. However, there research were not consistent in a sense that each type of random processes usually required a different approach and a separate analysis. Furthermore, in the particular case of Lévy processes analysed in this project, the research concentrated only on the one-dimensional case with only two exceptions, one of which was restricted to sets of bounded volume. Only recently in 2018 the first article which treated multidimensional isotropic stable Lévy processes in cones appeared.

The aim of our studies is to generalise already known results concerning the limiting conditioned distribution of multidimensional Lévy processes which are anisotropic, i.e. do not possess a friendly, rotation-invariant structure. In the first step we will focus on extending the result to some anisotropic stable processes. This class satisfies the *self-similarity* condition which means that due to the scaling property we may restrict ourselves to investigation of the process in one fixed moment in time. This property on one hand significantly simplifies work and on the other hand is appears frequently in mathematical descriptions of various physical and biological phenomena. In this task we will initially try to extend already known techniques but ultimately we plan to invent a new method based on the theory of convergence of stochastic processes.

In the second step we will investigate the *stability* of limits studied in the first part. Namely, it may happen that for certain processes which do not enjoy the self-similarity property anymore but are in some sense *similar* to the original one, the obtained Yaglom limit remains the same. Our task here will be to properly define the notion of *similarity* and then prove that the limiting conditional distribution is indeed the same. We expect that the key role in this step will be a proper rescaling of our process, since then under the assumption of its suitable *regularity* we should be able to deduce the desired similarity.

The outcome of our work will be a better knowledge and understanding of limiting behaviour of Lévy processes. We expect that our results will be of interest for scientists working not only in the field but also in applications of mathematics, for instance in its financial aspect. We also hope that the methods we plan to invent in this project will be applied in analysis of various probabilistic problems as well as in other areas of mathematics.