From expressive description logics to multi-variable fragments of first-order logic: reasoning in finite structures.

Searching for elegant logical formalisms with decidable satisfiability problem and reasonable expressive power is an important theme in computer science, with strong motivations coming from various areas, including hardware and software verification, artificial intelligence, distributed computing and databases.

Objects appearing in applications from these areas, e.g., databases, knowledge bases, computer programs or computations can be modelled as logical structures and their properties can be described by logical formulas. Then, algorithms solving *the satisfiability problem*, that is the problem of deciding whether a given input formula has a model, can be used to reason about the existence of objects meeting some given specification, e.g., to answer queries to data/knowledge bases or to inform about a possible dangerous behaviour of programs.

Of course there is a natural trade-off: the more properties can be specified in a given formalism (the richer the language is) the more difficult is to perform automatic reasoning. For example, a natural candidate for a specification language—first-order logic—while well established in mathematics and computer science and very elegant, is already too expressive for the considered applications as, since the classical works by Church and Turing in 1930s, it has been known that its satisfiability problem is undecidable, that is there is no algorithm (even in theory) that could solve it. In this project we are going to investigate potential specification languages belonging to two groups of formalisms: expressive *description logics* and multi-variable *fragments of first-order logic.*

Description logics have nowadays almost the status of an industrial standard, e.g., they form a basis for OWL, Web Ontology Language, and have successful applications in biomedicine and other fields. They are tailored for applications in Knowledge Engineering. Basic description logic (DL) notions are *concepts* (classes of objects) and *roles* (relations between objects). A specific DL is mainly characterized by the constructors it provides to build complex concepts and roles from atomic ones. A DL *knowledge base* (KB) consists of an ABox, which contains facts about individuals, and is a counterpart of a classical database, and a TBox, describing general background knowledge, from which additional facts, not explicitly present in the ABox can be inferred. Typical decision problem in this area is *KB satisfiability*: given a KB verify if it has a model (is consistent). Another problem studied is *ontology-mediated query answering*, OMQA. In this problem, apart from a KB, the input contains a *query.* The question is if the query is a consequence of the KB, that is, if the answer to the query is true in every model of the KB. Various classes of queries are considered. We will be interested in the class of *conjunctive queries* and in its generalizations.

We will focus on very expressive description logics from the families $\mathcal{Z}$ and $\mathcal{S}$, and will mostly work on the *finite* variants of satisfiability and OMQA, that is the variants in which the attention is restricted to finite models. This is a natural assumption, since objects arising in practical applications, like knowledge bases, are usually finite. While OMQA for important DLs is quite well understood, the study of finite OMQA has started only recently. In particular, a lot is to be done about finite OMQA in the family $\mathcal{Z}$ and investigating it is our first task.

DLs have their own specific syntax but most of them can be translated into first-order logic. Apart from studying DLs themselves one may thus also study fragments of first-order in which this translation can be embedded. Knowledge base satisfiability and OMQA translate then to satisfiability in appropriate fragments. While the translation can be done using only two variables, we will be mostly interested in fragments which do not bound the number of variables. Our main themes will be the recently introduced *triguarded fragment* and defined almost 50 years ago, slightly neglected since then, but, as we believe, worth reviving, *Maslov's class* $\bar{K}$. Studying fragments of this kind allows us to better understand the reasons behind decidability/undecidability of DLs, but it is also interesting on its own, as such fragments sometimes allow us to express properties not expressible in DLs, or even speak about things about which DLs cannot speak at all, like relations of arity greater than two. The last argument makes such fragments more suitable in some applications, e.g., in classical database theory.